MATH 163 —HOMEWORK # 10 : ANSWERS to some problems

Note: in most cases we DO NOT show all the work necessary to obtain the answer. You need to show all work.

Solutions Day 27:

1. (c) Use that \( \frac{k^2}{2k^{3/2} + 3} < \frac{k}{2k^{3/2}} = \frac{1}{2k^{3/2}} \). By Direct Comparison Test, since \( \sum 1/(2k^{3/2}) \) converges, so does the given series.

(d) Use that \( \frac{\ln(n)}{n+10} > \frac{1}{2n} \) for \( n \geq 10 \). By Direct Comparison Test, since \( \sum 1/(2n) \) diverges, so does the given series.

Solutions Day 28:

1. (b) Limit Comparison with \( \sum \frac{1}{j^{3/2}} \) shows convergence.

2. (d) Since \( \lim_{n \to \infty} a_n \neq 0 \), diverges by Divergence Test.

(f) Direct Comparison with \( \sum \frac{1}{n} \) shows divergence.

(g) Limit comparison with \( \sum 1/n^3 \) shows convergence.

3. \( a > 1 \). Why? Make sure you can explain.

4. (b) \( |c^3| < 1 \), so \( -1 < c < 1 \). Why? Make sure you can explain.

Solutions Day 29:

\[ \sum_{n=1}^{\infty} \frac{n}{n+2} \] diverges by Divergence Test (\( a_n \neq 0 \))

4. Converges by Alternating Series Test (AST). (As a preview to what we will do next week: This series does not converge absolutely by p-series. Therefore it is conditionally convergent, not absolutely.)

6. Converges by AST. Make sure to check all conditions, specially that the \( a_n \)s are decreasing. (Does not converge absolutely by Limit comparison with p-series. Therefore conditionally convergent.)

24. Converges by AST. (After the next section we will see that this series converges since it converges absolutely by Ratio Test. When applicable checking for absolute convergence is usually easier than applying the AST. It also gives you a stronger result.) The sum of the first 4 terms equals the infinite sum to within an error of 0.0001, since \( a_5 < 0.0001 \), but \( a_4 \) is not.

For all other problems, see book.