SOLUTIONS

Here we give the solutions to some of the Math 162 problems R1-R10, 100+.

REVIEW PROBLEMS

R1. (a) expression 1 is equal to expression 2

(b) equation 1 implies equation 2 (or equation 2 follows from equation 1) but the two equations are not necessarily the same

R3. (a) $-15$  (b) $\frac{93}{8}$  (c) $4$  (d) $3y^2/8x$  (e) $y^2/(x^2 + y^2)$  (f) $(x-7)/(2-x)$

(g) $\frac{3(3x-1)}{(x+3)(3x+1)}$  (h) $-3x^2 + 4 - \frac{2}{x^2 + 1}$

R4. (b) $-\frac{3}{7}x^2$

(c) $\frac{11}{x^{1/2}}$

(g) $x^{1/2} + 2x^{-1/2}$

R8. Odd: (e) since $f(-x) = \frac{-x}{-(x)} = \frac{-x}{x^2 + 1} = -f(x)$. Even: (a,b,d) (in each case, show that $f(x) = f(-x)$. Neither (c).

R10. (a) $f(x) = 2 \sin(5x)$ (Period = $2\pi/5$)

LIMITS

100. Make sure to show the limiting behaviour of the function on either side of the vertical asymptotes. You will lose points on an exam if you simply state the asymptote without showing work. (To obtain the figures below I used vertical asymptotes, horizontal asymptotes which we cover later, and intercepts.)

(b) Vertical asymptote at $x = 1$, with $\lim_{x \to 1^+} f(x) = \infty$, $\lim_{x \to 1^-} f(x) = -\infty$. Graph: below left.

(c) Vertical asymptote at $x = 3$, with $\lim_{x \to 3^+} f(x) = -\infty$, $\lim_{x \to 3^-} f(x) = +\infty$, and at $x = 4$, with $\lim_{x \to 4^+} f(x) = +\infty$, $\lim_{x \to 4^-} f(x) = -\infty$. Graph: below right.

102. (a) $55$  (c) DNE  (d) $0$  (g) $26/3$

(h) DNE ($\lim_{x \to 2^+} f(x) = \infty$, $\lim_{x \to 2^-} f(x) = -\infty$)

(l) $1$
105. Make sure to show the limiting behaviour of the function on either side of the vertical asymptotes. You will lose points on an exam if you simply state the asymptote without showing work.

(b) No vertical asymptotes. Horizontal asymptote y = 0. x-intercepts: none. y-intercepts: (0,1). Function is even. Graph sketched in Fig 1 below.

CONTINUITY

107. (a) The polynomial f(x) is continuous everywhere.

(b) The rational function H(x) is continuous at every point in its domain, that is, all \( x \neq (1 \pm \sqrt{21})/2 \)

(c) The rational function g(x) is continuous at every point in its domain, that is, at all \( x \in \mathbb{R} \).

DEFINITION OF DERIVATIVE

108. (b) (i) not continuous at \( x = 0 \) (vertical asymptote)
   (ii) not differentiable at \( x = 0 \) (not continuous) and \( x = 3 \) (cusp)
   (iii) the graph of \( f' \) has two vertical asymptotes at \( x = 0 \) and \( x = 3 \), since it looks as if the slopes go to either \( \infty \) or \( -\infty \) as these points are approached.

109. A function \( f(x) \) is differentiable at \( x = c \) if and only if \( f \) is defined at \( x = c \) and the limit

\[
\lim_{h \to 0} \frac{f(c + h) - f(c)}{h} \quad \text{or} \quad \lim_{x \to c} \frac{f(x) - f(c)}{x - c}
\]

exists. In that case, the derivative of \( f \) is defined as

\[
f'(c) = \lim_{h \to 0} \frac{f(c + h) - f(c)}{h}.
\]

111. (a) Make sure to plug in \( x = 0 \) from the beginning. That is, compute \( f'(0) \) using definition, not \( f'(x) \).

RULES FOR DIFFERENTIATION

112. (a) \( \frac{dR}{dx} = -\frac{7\sqrt{10}}{5x^3} \)

(b) \( f'(t) = \frac{1}{2^{1/3}} \left[ \frac{2}{3\sqrt{t}} - \frac{1}{6\sqrt[3]{t^2}} \right] \)

114. (a) \( f(x) = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases} \)
(b) By applying the rules in the interior of each subinterval we get \( f'(x) = \begin{cases} 
1 & \text{if } x \geq 0 \\
-1 & \text{if } x < 0 
\end{cases} \)

Note: we cannot apply the rules at an endpoint.
(c) Since \( \lim_{x \to 0^+} f'(x) = 1 \) and \( \lim_{x \to 0^-} f'(x) = -1 \) the function has a corner at the origin and is thus not differentiable there.
(d) The formula in (b) is the one for \( f'(x) \) (that is, we cannot assign a value for \( f' \) at 0. This is different than in the next problem.

117. (a) \( f'(t) = A\omega \cos(\omega t - \delta) \)
(b) \( g'(t) = a \sec(at) \tan(at) \)

119. (a) \( 2yy' \)
(b) \( 2yy'x + y^2 \)
(c) \( 6u^2u' - 9t^2 \)

121. \( P' = V'T + VT' \)

122. \( A = x^2 \Rightarrow \frac{dA}{dt} = 2x \frac{dx}{dt} \). When \( A = 16 \text{cm}^2 \) and \( \frac{dx}{dt} = 3 \text{cm/sec} \), then \( x = 4 \text{cm} \) and \( \frac{dA}{dt} = 12 \text{cm}^2/\text{sec} \).

127. (a) \( \frac{dV}{dh} = \frac{\pi r^2}{3} \)
(b) \( \frac{dV}{dr} = \frac{2\pi rh}{3} \)

129. (a) \( \frac{df}{dL} = -\frac{1}{2L^2} \sqrt{\frac{T}{\rho}} \), \( \frac{df}{dT} = \frac{1}{4L\sqrt{T\rho}} \), \( \frac{df}{d\rho} = -\frac{\sqrt{T}}{4L\sqrt{\rho^3}} \)

(b) When \( L \) increases the tension decreases since \( \frac{df}{dL} < 0 \). But when \( L \) decreases the tension increases since \( \frac{df}{dL} > 0 \)! (draw a graph)
When \( T \) increases the tension increases since \( \frac{df}{dT} > 0 \).
When \( \rho \) increases the tension decreases since \( \frac{df}{d\rho} < 0 \).

130. (a) 18ft
(b) \( H'(t) = 13.8 - 32t \). This is rate of change of height. Not rate of change of position, which has a nonzero horizontal component.
(c) Maximum height 20.976 ft at \( t = \frac{13.8}{32} \text{s} \)
(d) 6.900 ft/s
(e) \([0, \frac{13.8}{16}]\)
(f) 1.576s
(g) -36.639 ft/s
(h) -11.420 ft/s
(i) \( t = 0.7881 \text{s} \)

INVERSES

132. (a) Since \( f(0) = 3 \), it follows that \( f^{-1}(3) = 0 \).
\( (f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{5x^4 + 6x^2 + 2|_{x=f^{-1}(3)=0}} = \frac{1}{2} \)
(b) \( f^{-1}(1) = 0 \) since \( f(0) = 1 \) (use trial and error). Since \( f'(x) = 1 - \sin x \), have Since \( (f^{-1})'(1) = \frac{1}{1-\sin 0} = 1 \).

133. \( y = 2 + 7(x - 3) \)

**EXPONENTIALS AND LOGARITHMS**

134. (a) \( 2\sqrt{3} \)
(b) \( 2\pi \)
(c) \( 11 \)
(d) \( 4e\pi \)
(e) \( 2^{3\sqrt{3}+2\sqrt{7}-\pi} \)
(f) \( e^{\sqrt{2}} \)
(g) \( (4^7/25)^x \)
(h) \( (102, 400)^x \)
(i) \( (0.6)^x + (0.8)^x \)

135. (b,c) see figure

(e) To obtain figure use the fact that \( 2^{|x|} = \begin{cases} 2^x, & x \geq 0 \\ 2^{-x}, & x < 0 \end{cases} \)

136. (a) \(-1\) (Remember to divide by highest power in denominator, which one is it?)
(b) \(0\)
(c) \(0\)
(d) \(\infty\)
(e) \(\infty\)
(f) \(0\)

137. (a) No vertical asymptote. Since \( \lim_{x \to \infty} e^{-2x} \cos x = 0 \) and \( \lim_{x \to -\infty} e^{-2x} \cos x \) DNE (why not?), it follows the function has one horizontal asymptote \( y = 0 \).
(b) Vertical asymptote at \( x = e \) (check limiting behaviour on either side!) Horizontal asymptote \( y = 3 \) (compute limits!)
(c) Remember to divide by highest power in denominator to compute limits as \( x \to \pm\infty \).
(d) Again, divide by highest power in denominator to compute limits as \( x \to \pm\infty \). Note that since \( e^{7x} > 0 \) and \( e^{-7x} > 0 \) the denominator never equals zero so there are no vertical asymptotes.

138. (a) \( \frac{d}{dx}[\sinh x] = \cosh x \), \( \frac{d}{dx}[\cosh x] = \sinh x \). (Make sure you can show this!)

**L'HÔPITAL's RULE. RELATIVE RATES OF GROWTH**

140. Make sure you write the limit symbol until you actually take the limit, for example see solution 1a,e. As always, make sure you show all work.
(a) $\lim_{x \to 1} \frac{x^2 - 1}{x^2 - x} \cdot \left( \frac{0}{0} \right) = \lim_{x \to 1} \frac{2x}{2x - 1} = 2$ (used L'Hôpital's once)

(b) 1

(c) 0

(d) 0

(f) $\lim_{x \to 0} \frac{x + 1 - e^x}{x^2} \cdot \left( \frac{0}{0} \right) = \lim_{x \to 0} \frac{1 - e^x}{2x} \cdot \left( \frac{0}{0} \right) = \lim_{x \to 0} \frac{-e^x}{2} = -\frac{1}{2}$ (used L'Hôpital's twice)

(g) 0 (use L'Hôpital's three times)

(h) $+\infty$ (no L'Hôpital's needed)

(j) 0 (no L'Hôpital's needed)

(l) 0 (can either apply L'Hôpital’s three times, or first evaluate limit of term in brackets using L'Hôpital’s once)

(m) 0

(n) $-\infty$

(q) $\lim_{n \to \infty} (1 + r/n)^n = \lim_{n \to \infty} e^{n\ln(1 + r/n)} = e^r$ since the limit of the exponent is

$$\lim_{n \to \infty} n \ln(1 + r/n) = \lim_{n \to \infty} \frac{\ln(1 + r/n)}{1/n} \cdot \left( \frac{0}{0} \right) = \lim_{n \to \infty} \frac{\frac{1}{1+r/n}(-r/n^2)}{-1/n^2} = \lim_{n \to \infty} \frac{r}{1 + r/n} = r$$

(r) 1

141. (a) $x^n < e^x < x^x$. One limit needed is $\lim_{x \to \infty} e^{\frac{e^x}{x^n}} = \lim_{x \to \infty} e^{\frac{\infty}{\infty}} = \lim_{x \to \infty} \frac{e^x}{n(n-1)(n-2)\ldots2\cdot1} = \infty$ (here we applied L'Hôpital’s rule $n$ times). Therefore $x^n < e^x$.

(b) $\ln x < x < x^2 < x^3$. One limit needed is $\lim_{x \to \infty} \frac{x^3}{x^2} = \lim_{x \to \infty} x = \infty$ (here we used algebra, and not L'Hôpital’s). Therefore $x^2 < x^3$.

MAXIMA/MINIMA. CONCAVITY. GRAPHING.

142. (a) Local max = abs max= $|A|$, local min = abs min = $-|A|$

(b) Local max=2, no local min (use graph of parabola)

(c) Local max=1/$a^2$ (where denominator is smallest), no local min (denominator does not achieve finite max)

(d) Use that $f(x) = \frac{1}{x^{|x|}}$. Local max=1 (where denominator is smallest), no local min (denominator does not achieve finite max)
143. (b) (i) \( f'(x) = 12x^2(x - 1), \) \( x_c = 0, 1, \) increasing on \((1, \infty),\) decreasing on \((-\infty, 0) \cup (0, 1)\) (see signchart). Absolute min \( f(1) = 1 \) (at \( x = 1).\)

(ii) \( f''(x) = 12x(3x - 2) = 0 \) at \( x = 0, 2/3, \) concave up \((-\infty, 0) \cup (2/3, \infty),\) concave down \((0, 2/3)\) (see signchart). Inflection points \((0, 2), (2/3, 38/27)\)

(c) (i) \( f'(x) = \frac{-2x}{(x^2 + 1)^2} = 0 \) at \( x = 0.\) From sign chart: increasing on \((-\infty, 0),\) decreasing on \((0, \infty),\) local max at \( M(0,1).\)

(ii) \( f''(x) = 2\left[\frac{3x^2 - 1}{(x^2 + 1)^2}\right] = 0 \) at \( x = \pm 1/\sqrt{3}.\) From sign chart: concave up \((-\infty, -1/\sqrt{3}) \cup (1/\sqrt{3}, \infty),\) concave down \((-1/\sqrt{3}, 1/\sqrt{3}).\) Inflection points at \( I_1,2(\pm 1/\sqrt{3}, 0.75).\)

(Since the denominator is \( > 0,\) for sign charts it is enough to check sign of top, a polynomial in both cases.)

(iii) no x-intercept, y-intercept \((0,1),\) horizontal asymptote \( y = 0,\) even

145. Answer: not necessarily. \( f \) is increasing at \( x = 2 \) but could have a local max between 2 and 2.0000001 to the right of which it decreases such that \( f(2.0000001) < f(2).\)

148. (Note: answer in back of book is hopelessly incomplete.)

First note that \( y = -Ax^2(x - L)^2, \) where \( A = W/(24EI) \) is a negative constant if \( W < 0,\) and positive if \( W > 0.\) Also note that the domain is \( 0 \leq x \leq L.\) To figure out local maxima/minima and inflection points consider, for simplicity,

\[
f(x) = x^2(x - L)^2
\]

This function is a quartic with positive leading coefficients and double roots at \( 0, L, \) so a rough draft of the graph must look like sketch at right. The local maximum is at \( x = L/2, \) which follows using that \( f'(x) = 2(x^2 - Lx)(2x - L).\)

From \( f''(x) = 2[6x^2 - 6xL + L^2] = 0 \) we deduce that there are two inflection points at \( x = L/2 \pm L/\sqrt{12}. \) (From our first rough draft given by the sketch we already know these must be inflection points, so no need to check sign of second derivative on intervals.) Using the fact that \( f(L/2) = L^4/16, \) \( f(L/2 \pm L/\sqrt{12}) = L^4/36, \) restricting to the domain \( x \in [0, L],\) and multiplying graph by \(-A,\) we obtain graphs below:
ANTIDERIVATIVES. DIFFERENTIAL EQUATIONS.

150. ALWAYS check your result by differentiating your antiderivative.

(a) \( \frac{x^3}{3} - 5x^2/2 + 2x + C \)

(b) \( 2 \sec x + 2x^{1/2} + C \)

(c) Try \( e^{2t} \) and then adjust with a constant factor to obtain that the antiderivative of \( 4e^{2t} \) is \( 2e^{2t} + C \).

(d) \( \int 3 \cos(4x) + 2x \, dx = \frac{3}{4} \sin(4x) + x^2 + C \) (again, strategy is guess and check)

(e) \( \frac{1}{12} (3s - 2)^4 + C \)

(f) Since \( x^2(\sqrt{x} + 1)^2 = x^3 + 2x^2 \sqrt{x} + x^2 \), its antiderivative is \( \frac{1}{4} x^4 + \frac{4}{7} x^{7/2} + \frac{1}{3} x^3 + C \)

(g) Since the antiderivative of the \( \sec^2 x \) is \( \tan x \) we try \( \tan(8s) \) and then adjust with a constant factor to obtain that the antiderivative of \( \sec^2(8s) \) is \( \frac{1}{8} \tan(8s) + C \). ALWAYS CHECK that the derivative of the antiderivative is the right thing.

(h) \( \frac{2}{5} t^{5/2} + \frac{2}{3} t^{3/2} + 6t^{1/2} + C \)

(i) Since \( x(x + 1)(x + 2) = x^3 + 3x^2 + 2x \), its antiderivative is \( \frac{x^4}{4} + x^3 + x^2 + C \)

(j) \( \ln |x - 2| + C \)

(k) \( \frac{1}{103} 3x - \frac{1}{x} - \ln |x| + C \)

151. (a) \( g(x) = x^2 + 3x + C \)

(b) \( g(x) = x^2 + 3x - 2 \)

(c) \( f(t) = \sin(2t) + \tan(t) + 4 - 3 \sqrt{3}/2 \)

(d) \( h(t) = 3e^{2t} + C \). Since \( h(0) = 3 + C = -1 \) it follows that \( C = -4 \) and thus \( h(t) = 3e^{2t} - 4 \).
(e) \( f(x) = \frac{3}{5}x^{4/3} - \sin x + Cx + D \)

152. (a) \( \frac{2}{\sqrt{\pi}} e^{-x^2} \)
(b) where the derivative is positive: everywhere, all \( x \in \mathbb{R} \)
(c) concave up where the derivative is increasing: \( x \in (-\infty, 0) \)
concave down where the derivative is decreasing: \( x \in (0, \infty) \)

157. Given: \( a(t) = -5.3064 \text{ ft/sec}^2 \) and \( v(0) = 0 \) (since object dropped). Here distance axis is pointing upwards (so that acceleration is negative). Choose coordinate axis such that \( s(0) = 0 \).
Want: to know \( s \) after 5 seconds.
Solution:
antidifferentiate \( a \) to obtain \( v(t) = -5.3064t + c \). Use \( v(0) = 0 = -5.3064 \cdot 0 + c = c \) to determine that \( c = 0 \). Thus \( v(t) = -5.3064t \). Antidifferentiate \( v \) to obtain \( s(t) = -5.3064t^2/2 + d \). Use \( s(0) = 0 = -5.3064 \cdot 0^2/2 + d = d \) to determine that \( d = 0 \). Thus \( s(t) = -5.3064t^2/2 \). Now find that \( s(5) = -5.3064 \cdot 25/2 \approx -66.33 \) ft. The cliff is approximately 66.33 ft high.

158. Maximal height \( s\left(\frac{45\sqrt{3}}{32}\right) = 27075/64 \approx 423.05 \) ft.
Velocity at height 50 \( \approx v(0.3134) \approx 154.5 \) ft/s.

159. No, he misses hitting it by just a little bit. Total distance travelled by train until he stops: 9229 ft

**DEFINITE INTEGRAL**

160. (a) \( 73/6 \)
(b) \( 22 + 10\sqrt{3} \)
(c) \( 2 \cdot 98 \)
(d) \( 3(1000)(1001)/2 = 1,501,500 \)
(e) \( 2 \sum_{m=1}^{100} m^2 + 3 \sum_{m=1}^{100} m + 4 \sum_{m=0}^{100} = 2 \cdot \frac{100 \cdot 101 \cdot 201}{6} + 3 \cdot \frac{100 \cdot 101}{2} + 4 \cdot 101 \)
(f) \( 2 \cdot \frac{N(N+1)(2N+1)}{6} + 3 \cdot \frac{N(N+1)}{2} + 4(N + 1) \)

161. (a) \( \sum_{j=4}^{8} \frac{1}{j} \)
(c) \( \sum_{j=2}^{5} \frac{j}{2j+1} \)

162. By writing them out
\[ \sum_{j=1}^{N} j^2 = 1 + 2 + 4 + 9 + 25 + \ldots + N^2 \]
\[ \sum_{j=1}^{N^2} j = 1 + 2 + 3 + 4 + 5 + 6 + \ldots + N^2 \]
it is clear that the second sum is larger.

163. Approximations
\[ \begin{array}{c|c|c|c}
\text{n} & \text{right endpoint} & \text{left endpoint} & \text{midpoint} \\
\hline
\end{array} \]
The midpoint rule seems to converge fastest. I expect the exact answer to be 13.33 with 4 digits of accuracy (since this is the value of the last two results using the midpoint rule). Using only results with \( n = 2, 4, 8 \), may expect exact answer to be 13 with 2 digits of accuracy.

165. (a) \[ \lim_{N \to \infty} \sum_{j=1}^{N} (1 + \frac{x_j^2}{j}) \Delta x, \] where \( x_j = -1 + j \Delta x \) (I chose right endpoint), \( \Delta x = \frac{1}{N} \)

(b) \[ \lim_{N \to \infty} \sum_{j=1}^{N} (1 + \left(-1 + \frac{4j}{N}\right)^2) \Delta x \]

\[ = \lim_{N \to \infty} \frac{4}{N} \sum_{j=1}^{N} \left(1 + 1 - \frac{8j}{N} + \frac{16j^2}{N^2}\right) \]

\[ = \lim_{N \to \infty} \frac{4}{N} \left[ \sum_{j=1}^{N} 2 - \frac{8}{N} \sum_{j=1}^{N} j + \frac{16}{N^2} \sum_{j=1}^{N} j^2 \right] \]

\[ = \lim_{N \to \infty} \frac{4}{N} \left[ 2N - \frac{8}{N} \frac{N(N+1)}{2} + \frac{16}{N^2} \frac{N(N+1)(2N+1)}{6} \right] \]

\[ = 4 \lim_{N \to \infty} \left[ 2 - \frac{8(N+1)}{2N} + \frac{16(N+1)(2N+1)}{6N^2} \right] \]

\[ = 4 \left(2 - 4 + \frac{16}{3}\right) = \frac{40}{3} \]

Pretty close to the approximations in problem Day 33, #4!

166. Since \( f \geq 0 \) on interval, integral = area = 40/3.

167. Using right endpoint rule: let \( x_j = -1 + j \Delta x, \) \( j = 1, \ldots, N, \) \( \Delta x = 6/N, \) then

\[ \int_{-1}^{3} (1 + 3x) \, dx = \lim_{N \to \infty} \sum_{j=1}^{N} (1 + 3x_j) \Delta x = \lim_{N \to \infty} \frac{6}{N} \sum_{j=1}^{N} \left(-2 + 18\frac{j}{N}\right) \]

\[ = \lim_{N \to \infty} \frac{6}{N} \left(-2N + 18\frac{N(N+1)}{2N}\right) \lim_{N \to \infty} 6(-2 + 9\frac{(N+1)}{N}) = 6(-2 + 9) = 42 \]

which agrees with what you get by interpreting the integral as the difference of two areas (check it! be able to do this problem both ways).

168. (a) \[ \lim_{n \to \infty} \sum_{j=1}^{n} \left(\frac{2j}{n}\right)^3 \frac{2}{n} \]
169. (a) \( \int_{-\pi}^{\pi} \cos(x^2) \, dx \)
(b) \( \int_{0}^{1} \sqrt{1 + x} \, dx \)

171. 124.1644

172. \( F(x) = \begin{cases} 
\frac{x^2}{2} & \text{if } x \leq 3 \\
\frac{9}{2} + 3(x - 3) & \text{if } x > 3 
\end{cases} \)

173. increasing on \((0, 1.5) \cup (3, 4)\)

decreasing on \((1.5, 3)\)

Graph: \( F(x) = \) area between \( t = 0 \) and \( t = x \), starts out at 0 when \( x = 0 \), then increases until \( x = 1.5 \), then starts decreasing until \( F(3) = 0 \) (from apparent symmetry). Then \( F \) starts increasing again.

175. Do not find \( F \). Use that \( F'(x) = x(x - 1) \), and \( F''(x) = 2x - 1 \), to get: increasing on \((-\infty, 0) \cup (1, \infty)\), decreasing on \((0, 1)\), concave up on \((\frac{1}{2}, \infty)\), concave down on \((-\infty, \frac{1}{2})\).

176. Note \( F'(x) = f(x) \). Thus \( f(x) = \begin{cases} 
2 & \text{if } 0 < x < 1 \\
0 & \text{if } 1 < x < 2 \\
-1 & \text{if } 2 < x < 3 
\end{cases} \)

Notice that the function \( f \) is not continuous in the whole interval \([0,3]\) but that it has a finite number of jumps and in fact is a piece-wise defined function continuous on each of the three subintervals. In that case the integral of \( f \) is still well defined.

177. Hint: what happens if you differentiate the equation?

\textbf{INDEFINITE INTEGRAL}

180. (a) \( t^2/2 + 2\sqrt{3}t^{3/2}/3 - \ln |t| + C \)
(b) \( -\cos(2t)/2 + C \)
(c) \( 2t - t^2 + t^3/3 - t^4/4 + C \)
(f) \( \sec(\pi x)/\pi + C \)

181. The result follows from the fact that \( p(t) \) is an antiderivative of \( v(t) \).

182. (a) \( 8300/3 \)
(b) \( v > 0 \) at \( t \in (2, 10) \), \( v < 0 \) at \( t \in (0, 2) \)
(c) \( |p(2) - p(0)| + |p(10) - p(2)| = 8340/3 \)