

Find the eigenpairs of $A = \begin{pmatrix} 3 & -1 \\ 4 & -2 \end{pmatrix}$.

We have $A\bar{x} = \lambda\bar{x}$.

STEP 1 $(A - \lambda I)\bar{x} = 0 \xrightarrow{\text{to have nonzero } \bar{x}} \det(A - \lambda I) = 0$

$$A - \lambda I = \begin{pmatrix} 3-\lambda & -1 \\ 4 & -2-\lambda \end{pmatrix} \Rightarrow \det(A - \lambda I) = \boxed{(3-\lambda)(-2-\lambda) + 4 = 0}$$

characteristic eqn. for A

$$\Rightarrow \lambda^2 - \lambda - 2 = 0 \Rightarrow \lambda_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \Rightarrow \begin{cases} \lambda_1 = 2 \\ \lambda_2 = -1 \end{cases}$$

STEP 2 Let $\lambda_1 = 2 \Rightarrow A\bar{x} = \lambda_1\bar{x} \Rightarrow \begin{pmatrix} 3 & -1 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 3-2 & -1 \\ 4 & -2-2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} x_1 - x_2 = 0 \\ 4x_1 - 4x_2 = 0 \end{cases} \Rightarrow \boxed{x_1 = x_2}$$

There are infinitely many solutions. We pick one, for example

we pick $x_1 = x_2 = 1$.

Hence, the eigenvector corresponding to $\lambda_1 = 2$, is $\bar{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Let $\lambda_2 = -1 \Rightarrow A\bar{x} = \lambda_2\bar{x} \Rightarrow$

$$\begin{pmatrix} 3+1 & -1 \\ 4 & -2+1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 4x_1 - x_2 = 0 \\ 4x_1 - x_2 = 0 \end{cases} \Rightarrow \boxed{4x_1 = x_2}$$

Again, there are infinitely many solutions. we pick one:

$x_1 = 1, x_2 = 4$.

Hence, the eigenvector corresponding to $\lambda_2 = -1$ is $\bar{x} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.