Theorems of ch. 2

T1 Consider $A = (a_{ij}) \in \mathbb{R}^{n \times n}$.

Let $M_{ij} \in \mathbb{R}^{(n-1)\times(n-1)}$ be the minor matrix of a_{ij}, obtained from matrix A by deleting row 1 and column j of A.

Let $m_{1j} = (-1)^{1+j} \cdot \det(M_{1j})$ be the cofactor of a_{1j}.

Then $$\boxed{\det(A) = a_{11} \cdot m_{11} + a_{12} \cdot m_{12} + \cdots + a_{1n} \cdot m_{1n}}$$

* proof is not required.

T2 Consider $A = (a_{ij}) \in \mathbb{R}^{n \times n}$.

Let $M_{ij} \in \mathbb{R}^{(n-1)\times(n-1)}$ be the minor matrix of a_{ij}, obtained from matrix A by deleting row i and column j of A.

Let $m_{ij} = (-1)^{i+j} \cdot \det(M_{ij})$ be the cofactor of a_{ij}.

Then $$\boxed{a_{i1} \cdot m_{j1} + a_{i2} \cdot m_{j2} + \cdots + a_{in} \cdot m_{jn} = \begin{cases} \det(A) & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}}$$

* proof is not required.

T3 Let $A \in \mathbb{R}^{n \times n}$. Then $\boxed{\det(A^T) = \det(A)}$.

* proof is not required.

T4 Let $A \in \mathbb{R}^{n \times n}$ be a triangular matrix. Then

$$\boxed{\det(A) = \text{product of diagonal elements of } A}.$$

* proof is not required.

$\boxed{T5}$ Consider $A = (a_{ij}) \in \mathbb{R}^{n\times n}$.

If A has a zero column or a zero row $\Longrightarrow$ $\det(A) = 0$

If A has two identical columns or two identical rows $\Longrightarrow$ $\det(A) = 0$

* proof is not required.

$\boxed{T6}$ Consider $A \in \mathbb{R}^{n\times n}$ and let $E \in \mathbb{R}^{n\times n}$ be an elementary matrix.

Then $\boxed{\det(EA) = \det(E)\cdot\det(A)}$.

where $\boxed{\det(E) = \begin{cases} -1 & \text{if } E \text{ is of type I} \\ \alpha \neq 0 & \text{if } E \text{ is of type II} \\ 1 & \text{if } E \text{ is of type III} \end{cases}}$

* proof is not required.

$\boxed{T7}$ $A \in \mathbb{R}^{n\times n}$ is singular $\underbrace{\text{iff}}_{\text{(if and only if)}}$ $\det(A) = 0$.

* proof is not required.

$\boxed{T8}$ Let $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times n}$. Then $\boxed{\det(AB) = \det(A)\cdot\det(B)}$.

* proof is not required.