Mohammad Motamed
MATH/CS 375, Spring 2015 – HW # **7**

UNM College of Arts & Sciences
Mathematics and Statistics

Hand in a report on Thursday April 21 in class.

Note: Do not include any MATLAB code in your report, unless for problems 1(b) and 1(c).

---

1. In this problem you find and evaluate the polynomial of degree $m \leq n-1$,

$$P_m(x) = a_m\, x^m + a_{m-1}\, x^{m-1} + \cdots + a_1\, x + a_0,$$

that interpolates n data points (x_i, y_i), with $i = 1, \ldots, n$.

   (a) Assume $(x_i, y_i), i = 1, \ldots, n$, are given. Derive the system $V_n\, \mathbf{a} = \mathbf{y}$ that determines the coefficient vector $\mathbf{a} = (a_0, a_1, \ldots, a_m)^\top$, where $V_n \in \mathbb{R}^{n\times n}$ is the Vandermonde matrix.

   (b) Use Vandermonde approach and write a MATLAB function `function a=interpvan(x,y)` that returns the vector of coefficients $\mathbf{a}$ given input vectors $\mathbf{x} = (x_1, x_2, \ldots, x_n)^\top$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)^\top$. List your MATLAB script.

   (c) Write a MATLAB function `function ynew=vaninterp(a,xnew)` that evaluates P and returns $\mathbf{y}_{\text{new}} = P(\mathbf{x}_{\text{new}})$. Here, $\mathbf{x}_{\text{new}}$ is a vector containing the x-coordinate of new points. List your MATLAB script.

   (d) Apply your functions in (b) and (c) to find the polynomial that interpolates

$$(-1,-1), (0,2), (2,0), (3,5), (5,6), (6,9), (8,12)$$

   and plot the polynomial on $x \in [-1.5, 8.5]$ together with the $n = 7$ data points. Also compute the condition number of the Vandermonde matrix in the infinity norm, $\kappa_\infty(V_7)$, and the residual $r = ||V_7\, \mathbf{a} - \mathbf{y}||_2$. You should state the polynomial, show the plot, and report κ_∞ and r.

   (e) Use the Newton's divided differences approach and write a MATLAB function called `newtoninterp` that takes input vectors $\mathbf{x}$, $\mathbf{y}$, $\mathbf{x}_{\text{new}}$ and returns $\mathbf{y}_{\text{new}} = P(\mathbf{x}_{\text{new}})$. Use the data points in part (d) and plot the polynomial on $x \in [-1.5, 8.5]$ together with the $n = 7$ data points. Also compute the residual $r = ||P(\mathbf{x}) - \mathbf{y}||_2$ and compare it with the residual obtained by Vandermonde approach in part (d).

2. In this problem you explore the Runge phenomenon that can occur in polynomial interpolation. Consider

$$f(x) = \frac{1}{1+16x^2}, \quad x \in [-1, 1]\ ,$$

and let $y_i = f(x_i)$, $i = 1, \ldots, n$, where the values x_i are given below:

   (a) Equally spaced points $x_i = -1 + 2(i-1)/(n-1)$, $n = 10, 20, 40$.

   (b) Chebyshev points $x_i = \cos\dfrac{(2i-1)\pi}{2n}$, $n = 10, 20, 40$.

Using the Lagrange polynomial approach, find and plot the interpolating polynomial to the n data points. Show all interpolants on separate plots (total of 6 plots), but use the same window $x \in [-1, 1], y \in [-0.5, 1.5]$ for all plots. Discuss your results.

3. In this problem you try piecewise polynomial interpolation techniques to interpolate a set of data points in two dimensions.

   First, use `ginput` to get $n = 10$ data points and plot them. (Try `help ginput` in MATLAB to learn how it works):

```
clf
axis([-1 1 -1 1])
[pts] = ginput;
x = pts(:,1);
y = pts(:,2);
plot(x,y,'r*')
```

   (a) **High order interpolation:** Use your functions from problem (1e) and find and plot the high order polynomial (of degree at most 9) that interpolates all the points. Your plot should show the polynomial together with the 10 data points.

   (b) **Natural spline:** Write a MATLAB code that takes the vectors $\mathbf{x}$ and $\mathbf{y}$, containing the 10 data points, and computes a natural cubic spline by solving a system of equations. Plot the spline together with the data points.

   Hand in some neat plots, compare your plots in parts (a) and (b), and discuss.