Stat 453/553: Assignments

HW 1: (due on February 7th Friday)

Note that for all the problems related to finding pdf, you will need to specify the domain of the random variable)

- Textbook: 5.6, 5.10c, 5.15, 5.16, 5.17cd, 5.21, 5.24, 5.27
- Problem 1: Let X have the density $f(x) = \frac{2}{9}(x+1)$, $-1 \le x \le 2$. Find the density of $Y = X^2$
- Problem 2: Consider the following joint density function

  $$f(x, y) = 8xy, \quad 0 < y < x < 1.$$

  Let $U = X + Y$ and $W = X - Y$. Find the joint pdf of U and W.
- Problem 3: Consider a random sample of size n from a distribution with pdf and CDF given by $f(x) = 2x$ and $F(x) = x^2; 0 < x < 1$. Let $R = X_{(n)} - X_{(1)}$ be the range of the sample.

  (1) Give a general form of the density function of R

  (2) Find the density function of R when $n = 2$.

HW 2: (due on February 19th Wed)

- Textbook: 5.29, 5.34, 5.43(a), 5.44
- Problem 1: 1.Assume that $X_1, X_2, \ldots, X_n$ denote a random sample from a population with the following probability density function :

  $$f_X(x|\alpha) = \frac{\alpha\beta}{(\alpha+\beta x)^2},\ x > 0$$

  where $\alpha > 0$ and $\beta > 0$.
  find the limiting distribution of $n\beta X_{(1)}$.
- Problem 2: Assume that $X_1, X_2, \ldots, X_n$ denote a random sample from a poisson population with parameter λ. If the limiting distribution exists, $\sqrt{n}(\sqrt{\bar{X}_n} - \sqrt{\lambda}) \to N(0, C)$, find C.
- Problem 3: Suppose $X_1, \ldots, X_n$ are a random sample from the $Gamma(\alpha, \beta)$ for unknown $\alpha > 0$ and $\beta > 0$.
  Find c and d such that

  $$\sqrt{n}\left(\frac{n}{\sum_{i=1}^{n} X_i} - c\right) \xrightarrow{d} N(0, d).$$

HW 3: (due on Mar 4th Wed)

- Textbook: 6.9, 6.12, 6.17
- Problem 1: Let $X_1, \ldots, X_n$ be iid observations from a pdf or pmf that belongs to an exponential family given by

  $f(x|\theta) = h(x)c(\theta)exp(\sum_{i=1}^{k} w_i(\theta)t_i(x))$,

  find a sufficient statistic for θ
- Problem 2. Let $X_1, \ldots, X_n$ be independent random variables with pdfs

$$f(x_i|\theta) = \begin{cases} \dfrac{1}{2i\theta} & \text{if } -i(\theta-1) < x_i < i(\theta+1) \\ 0 & \text{otherwise} \end{cases}$$

  (a) Find a sufficient statistic for θ.

  (b) Is it a minimal sufficient statistic?
- Problem 3. Let $X_1, \ldots, X_n$ be iid variables with distribution $N(\theta, a\theta^2)$ where a is known and $\theta > 0$.

  (a) Show $T = (\bar{X}, S^2)$ is a sufficient statistic for θ.

  (b) Is it a minimal sufficient statistic?

  (c) Show it is not complete.
- Problem 4. Let X_1,..., X_n be an i.i.d. sample from a geometric distribution with parameter p. Define U as

$$U = \begin{cases} 1, & \text{if } X_1 = 1, \\ 0, & \text{if } X_1 > 1. \end{cases}$$

  (a) find $E(U)$.

  (b)find a sufficient statistic T for p.

  (c)Is U a sufficient statistic? Why?

  (d)find $E(U|T)$

HW 4: (due on Mar 25th Wed)

- Textbook: 6.20, 6.23, 6.31 (a)