Stat 345 Lecture Notes: Counting Formulas

Fundamental Principle of Counting (Multiplication Rule)

If an operation can be described as a sequence of k steps, and

if the number of ways of completing step 1 is n_1, and

if the number of ways of completing step 2 is n_2 for each way of completing step 1, and

if the number of ways of completing step 3 is n_3 for each way of completing step 2, and so forth,

then the total number of ways of completing the operation is

$$n_1 \times n_2 \times \cdots \times n_k$$

Permutations (ordered sequence of the elements)

n different elements can be lined up in $n!$ ways, where

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

Permutations of similar objects

n objects with $n = n_1 + n_2 + \cdots + n_r$, of which n_1 are of one type, n_2 are of a second type, $\cdots$, and n_r are of an rth type can be lined up in $\dfrac{n!}{n_1!n_2!n_3!\cdots n_r!}$ways.

Permutations of Subsets

The number of permutations of subsets of r elements selected from a set of n different elements is

$$P_r^n = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}$$

Combinations (order is not considered)

The number of combinations, subsets of size r that can be selected from a set of n elements is

$$\begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{r!(n-r)!}$$

Examples:

1. In the layout of a printed circuit board for an electronic product, there are 12 different locations that can accommodate chips.

(a) If five different types of chips are to be placed on the board, how many different layouts are possible?

(b) If the five chips that are placed on the board are of the same type, how many different layouts are possible?

2. A part is labeled by printing with four thick lines, three medium lines, and two thin lines. If each ordering of the nine lines represents a different label, how many different labels can be generated by using this scheme?

3. A lot of 140 semiconductor chips is inspected by choosing a sample of five chips. Assume 10 of the chips do not conform to customer requirements.

(a) How many different samples are possible?

(b) How many samples of five contain exactly one nonconforming chip?

(c) How many samples of five contain at least one nonconforming chip?