Chapter 4: Review Calculus

- Differentiation: Polynomial, e^x, chain rule, mutiplication rule, quotient rule
- Integration: Polynomial, e^x, substitution rule, integration by parts

Differentiation

- $\dfrac{d}{dx}(x^n) = nx^{n-1}, \dfrac{d}{dx}(c) = 0$

  Ex: $\dfrac{d}{dx}(x^3) = 3x^2$

- $\dfrac{d}{dx}(e^x) = e^x$

- $\dfrac{d}{dx}(f(x) \cdot g(x)) = f'(x)g(x) + f(x)g'(x)$

  Ex: $\dfrac{d}{dx}(x^2e^x) = 2x \cdot e^x + x^2 \cdot e^x$

- $$\frac{d}{dx}(\frac{f(x)}{g(x)}) = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

  Ex: $$\frac{d}{dx}(\frac{e^x}{x^2}) = \frac{e^x \cdot x^2 - e^x \cdot 2x}{x^4}$$

- $$\frac{d}{dx}\text{log}_a x = \frac{1}{x}\text{log}_a e$$

- $$\frac{d}{dx}\text{ln}(x) = \frac{1}{x}$$

- $$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$$

  Ex: $$\frac{d}{dx}(e^{x^2}) = e^{x^2} \cdot 2x$$

Integration

$$\int f(x) = F(x) + C, F'(x) = f(x)$$

$$\int_a^b f(x) = F(x)|_a^b, F'(x) = f(x)$$

- Polynomial $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

  Ex:

$$\begin{aligned} \int_0^1 x^5 dx &= \frac{x^6}{6}\Big|_0^1 \\ &= \frac{1}{6} - 0 \\ &= \frac{1}{6} \end{aligned}$$

- $\int e^x dx = e^x + C$

- Substitution $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

  Ex: $\int_0^1 (5x+4)^3 dx$

  Let $u = 5x+4$, $du = 5dx$, $dx = \dfrac{du}{5}$

$$\begin{aligned} \int_0^1 (5x+4)^3 dx &= \int_4^9 u^3 \frac{du}{5} \\ &= \frac{u^4}{4\times 5}\Big|_4^9 \\ &= \frac{9^4}{20} - \frac{4^4}{20} \\ &\approx 1.28 \end{aligned}$$

  practice: (1) $\int e^{-5x} dx$

  (2) $\int x e^{x^2} dx$

- Integral by parts

$$\int u dv = uv - \int v du$$

Ex: $\int xe^{-x}dx$

Let $u = x, v = -e^{-x}$

$du = dx, dv = e^{-x}dx$

$$\begin{aligned}
\int xe^{-x}dx &= \int udv \\
&= uv - \int vdu \\
&= -xe^{-x} - \int -e^{-x}dx \\
&= -xe^{-x} + \int e^{-x}dx \\
&= -xe^{-x} - e^{-x} + C
\end{aligned}$$