Adapted Autoregressive Model and Volatility Model with Application

Naisheng Wang, Financial Analyst, China Securities Index Co., Ltd, Shanghai, China, wangnaisheng@yahoo.com

Yan Lu, Assistant Professor, Department of Mathematics and Statistics, University of New Mexico, MSC03 2150, Albuquerque NM 87131, USA, luyan@math.unm.edu, 1-505-227-2544

ABSTRACT

Price limits are applied to control risks in various futures markets. In this paper, we proposed an adapted autoregressive model for the observed futures return with introducing dummy variables that represent limit moves. We also proposed a stochastic volatility model with dummy variables. These two models are used to investigate the existence of price delayed discovery effect and volatility spillover effect from price limits. MCMC method is used to estimate the parameters and to test hypothesis. We give an empirical study of the impact of price limits on copper and natural rubber futures in Shanghai Futures Exchange (SHFE) of China.

KEYWORDS: Autoregressive model, Price delayed discovery effect, Price limits, MCMC sampling, Stochastic Volatility model

1 Introduction

More than two thirds of the organized markets in the world have price limits (Hall & Kofman, 2001). Price limits describe the highest and lowest prices that a commodity or option is permitted to reach in a given trading session. It is an important regulation to control risks and to inhibit excessive fluctuation of futures price.

The effectiveness of price limits are usually examined according to price delayed discovery effect, volatility spillover effect and trading disturbance effect. When a limit is reached,

trading stops and the equilibrium price is not observed. Under hypothesis of price delayed discovery effect, price will continue to reach equilibrium price during the following trading days. Under hypothesis of volatility spillover effect, trading following a limit move will reflect unrealized fluctuations of that day. Therefore the period of price fluctuation extends to a longer one. Generally, if the hypotheses of price delayed discovery effect and volatility spillover effect are accepted, it is considered that price limits are useless.

In this paper, we proposed an autoregressive model for the observed futures return with introducing dummy variables that represent limit moves. We also proposed a stochastic volatility model with dummy variables. Using these two models, we investigate the existence of price delayed discovery effect and volatility spillover effect by studying copper and natural rubber futures in SHFE. Due to the difficulty of data collection, we didn't discuss trading disturbance effect. This paper consists of five Chapters. Chapter two discusses the realization of price delayed discovery effect and volatility spillover effect. Chapter three proposes testing models and estimating methods. Chapter four presents an empirical study of the effects of price limits in SHFE. Lastly, we give a conclusion in Chapter five.

2 The influence of price limits on futures price

In this section, first, we review futures return under price limits. We decompose the true equilibrium futures return in three parts: the observed return, the unrealized part of that trading day and the unrealized equilibrium return carried over from the previous trading day. Based on our decomposition, we discuss price delayed discovery effect. Then we discuss volatility spillover effect.

2.1 Futures return under price limits

In a market with a daily price limit, trading is permitted only at prices within limits determined by the settlement price of the previous day. In Chinese futures markets, the absolute variation is 3% around settlement price of the previous day under normal situation. If the bid is outside the allowed trading range, it is considered invalid. Therefore if equilibrium price moves outside the limits, price limits are the price we observe. The relationship between the observed futures price and equilibrium price at t^{th} trading day can be described as the following:

$$F_t = \begin{cases} F_{t-1}(1+l) & if\ F_t^e \geq F_{t-1}(1+l) \\ F_t^e & if F_{t-1}(1-l) < F_t^e < F_{t-1}(1+l) \\ F_{t-1}(1-l) & if F_t^e \leq F_{t-1}(1-l) \end{cases} \quad (1)$$

Where F_t^e is the equilibrium price at time t, F_t is the observed price and l is the maximum daily limit imposed on the absolute change in futures price within a trading day. In Chinese futures markets l = 3%. The observed futures price is equal to the true equilibrium price only if the futures equilibrium price falls within the price limits. If the futures price we observe is equal to the limits, the true equilibrium price must be higher (or lower) than the price observed.

The Log returns $log\ (F_t/F_{t-1})$ is commonly used during empirical study. It can be written as the following: .

$$r_t = \begin{cases} r^u & if\ F_t^e > F_{t-1}(1+l) \\ \ln(F_t^e / F_{t-1}) & if F_{t-1}(1-l) < F_t^e < F_{t-1}(1+l) \\ r^d & if F_t^e < F_{t-1}(1-l) \end{cases}, \quad (2)$$

Where $r_t = F_t / F_{t-1}$, $r^u = \ln(1+l)$, and $r^d = \ln(1-l)$. r^u and r^d are the limit-up and limit-down of futures log return respectively. We can see that the observed return is not necessarily equal to the true equilibrium return. This is determined by the fact whether futures price of the previous trading day reaches the limits or not.

2.2 Price delayed discovery effect

Chou and Wu (1998) decomposed the equilibrium return as the sum of the observed return and the unrealized parts from the current trading day. They didn't consider the part carried over from the previous trading day. Therefore their conclusion is not suitable for the situation of consecutive limit hits.

In the following, we decompose the true equilibrium futures return as three parts:

$$\begin{aligned} r_t^e &= \ln(F_t^e / F_{t-1}^e) \\ &= \ln(F_t / F_{t-1}) + \ln(F_t^e / F_t) - \ln(F_{t-1}^e / F_{t-1}) \\ &= r_t + E_t - E_{t-1} \end{aligned} \tag{3}$$

Where $E_t = \ln(F_t^e / F_t)$ denotes the unrealized part of equilibrium return at the tth trading day because of the existence of price limits. From (3), we see that equilibrium return is equal to the sum of the observed return and the unrealized part of that trading day subtracting the unrealized equilibrium return carried over from the previous trading day. Particularly, if there is no limit move at $(t-1)^{th}$ trading day, $E_{t-1} = 0$; if there is no limit move

at tth trading day, $E_t = 0$. Otherwise, the unrealized part of equilibrium return will be carried over to the next trading day. This is called price delayed discovery effect (see Figure 1).

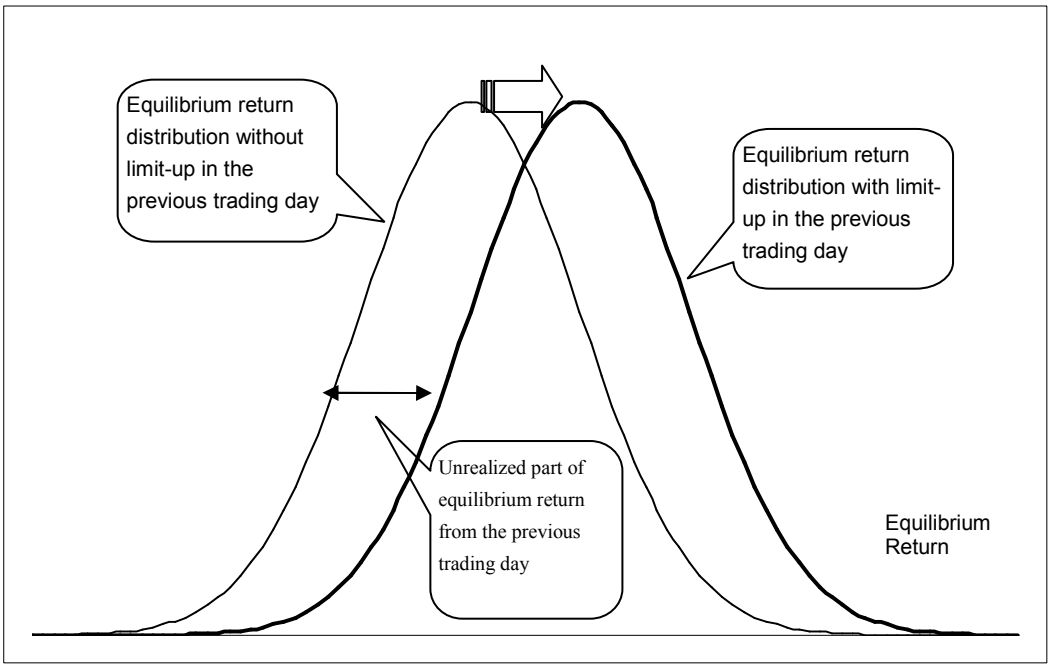

Figure 1

The distribution on the right is the futures equilibrium return of the day following a limit-up. The unrealized part of equilibrium return from the previous trading day is carried over to the next trading day.

If price reaches the upper limit at the tth-trading day, the estimated equilibrium return with price limits at that day is:

$$\hat{r}_t^e = E[r_t^e \mid r_t^e > r^u] \tag{4}$$

(4) is the conditional expected return given the equilibrium return greater than the limit-up return (see Figure 2).

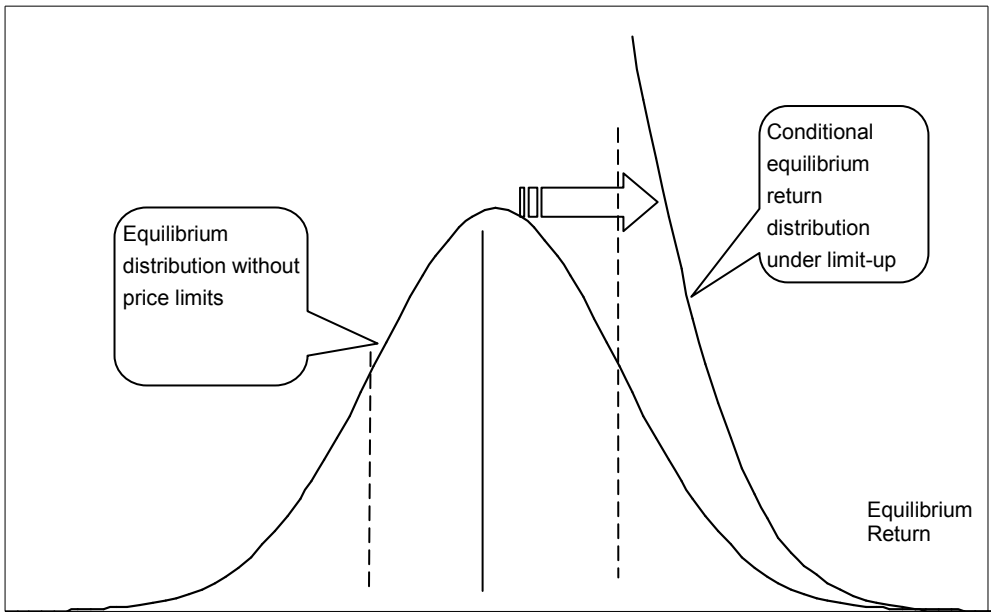

Figure 2

In Figure 2, between the two straight lines, we see the distribution under the price limits. The curve on the right indicates the conditional equilibrium return distribution under limit-up.

The equilibrium return of the next trading day is:

$$\hat{r}_{t+1}^{e}\,|_{r_t^e} = E[r_{t+1}^{e} \mid r_t^e > r^u] \ . \tag{5}$$

Suppose equilibrium return follows the simple stochastic model:

$$r_t^e = \mu + \varepsilon_t \,, \tag{6}$$

Where ε_t is a sequence of independent, identically distributed, random variables with mean 0 and variance $\sigma_t^{\,2}$. By (4) and (5) we obtain (7):

$$\begin{aligned} E[r_{t+1}^{e} \mid r_t^e > r^u] &= 2\mu + \phi(\Xi_t)\big/\big[1-\Phi(\Xi_t)\big] - r^u \\ &\geq \mu = E[r_{t+1}^{e}] \end{aligned} \tag{7}$$

Where $\Xi_t = (r^u - \mu)/\sigma_t$. (7) Indicates that futures price tends to go up following a limit-up day. On average, equilibrium return is greater than the equilibrium return without the enforcement of price limits. This is due to the price delayed discovery effect. Similarly, the futures price following a limit-down day tends to go down. Expected equilibrium return is lower than the expected equilibrium return without price limits. The same conclusion can be derived even we replace (6) by a more complicated model such as autoregressive processes. Trading stops when price reaches the limits. Under hypothesis of price delayed discovery effect, futures price will continue to go up or fall to reach equilibrium price in the following trading days, at least the next trading day.

2.3 Volatility spillover effect

Consider model (6) again. When price reaches the upper limit at the tth day, variance of conditional equilibrium return of the next trading day is given as the following:

$$\begin{aligned} Var[r_{t+1}^{e} \mid r_{t}^{e} > r^{u}] &= \sigma_{t+1}^{2} + \sigma_{t}^{2}\left\{1 - \frac{\phi(\Xi_{t})}{1-\Phi(\Xi_{t})}\left[\frac{\phi(\Xi_{t})}{1-\Phi(\Xi_{t})} - \Xi_{t}\right]\right\} \\ &\geq \sigma_{t}^{2} = Var[r_{t}^{e}] \end{aligned} \tag{8}$$

The result is similar even with a more complicated model. Because of the high volatility at the limit-hit-day, large fluctuation will continue in the following days. This indicates that futures return sequence is volatility clustering. Price fluctuation of the next trading day following a limit-hit-day will be larger than the fluctuation without price limits. This is called volatility spillover effect.

Volatility spillover effect and price delayed discovery effect is highly related. The unrealized part of the equilibrium return because of price limits will deliver to the next trading day. This part of return is uncertain. Consequently, the uncertainty of return of the following trading day will increase, resulting in fluctuation increscent. Price limits impede price fluctuation that should be completed within one trading day. This results in a longer period of price fluctuation and volatility spillover. As a result, price limits could prevent price slump and price jump, could reduce risk and could make markets more rational.

3 Testing hypothesis and methods

In this section, we discuss how to test price delayed discovery effect and volatility spillover effect. In literature, many researchers adopt case study. They investigate the effect of price limits by comparing futures return and fluctuation before the limit move to those after the limit move. These methods work well for stock markets, where a lot of limit moves are observed and different stocks can be studied together. However, it is not efficient in futures markets, where a single futures product is unlikely to have lots of limit moves and different futures are not supposed to study together because of different features. For example, there are only about 20 limit moves of copper futures in SHFE within four years (1999-2003). A futures product has several contracts at the same time, but they may have similar behavior because of arbitrage. If we consider these contracts together, it is actually duplicate computations on the same price. Furthermore, case study is hard to deal with the phenomena of consecutive-limit moves. Therefore, we propose a means model for the observed futures return with introducing dummy variables that represent limit moves. We also propose a stochastic volatility model with dummy variables. MCMC sampling method is used to estimate the parameters and test hypothesis.

3.1 Adapted autoregressive model

Generally we assume that the equilibrium return sequence is an autoregressive process (AR(1)), i.e

$$r_t^e = u + \varphi \ r_{t-1}^e + \varepsilon_t ,$$

Where, (μ, φ) are parameters, ε_t is the error term with mean zero. MCMC method and EM algorithm can be used to analyze the above model (Wei, 2002). There are some other researchers who consider dummy variables that reflect price limits in the above model (Chou & Wu, 1998). We believe the true equilibrium return is inherent, which would not be

affected by the existence of price limits. What has been affected is the observed return. As a result, in order to examine if price limits have impact on price delayed discovery effect, it is not appropriate to introduce dummy variable in equilibrium return model. We need to construct a model for the observed return and introduce dummy variable. The model we proposed is as follows:

$$r_t = u + \varphi\ r_{t-1} + \gamma\delta_{t-1}{}^{u} + k\delta_{t-1}{}^{d} + \varepsilon_t \tag{9}$$

$$\varepsilon_t \sim N(0, \sigma_t{}^2)$$

Where σ_t is volatility; $\delta_{t-1}{}^{u}$ and $\delta_{t-1}{}^{d}$ are the dummy variables reflecting whether there is a limit up or a limit down respectively. They are given as follows:

$$\delta_{t-1}^{u} = \begin{cases} 1, & r_{t-1} = r^u \\ 0, & r_{t-1} < r^u \end{cases}, \qquad \delta_{t-1}{}^{d} = \begin{cases} 1, & r_{t-1} = r^d \\ 0, & r_{t-1} > r^d \end{cases}$$

Following a limit up day, price return will move from $\mu+\varphi r_{t-1}$ to $\mu+\varphi r_{t-1}+\gamma$; following a limit down day, price return will move from $\mu+\varphi r_{t-1}$ to $\mu+\varphi r_{t-1}+k$. The null hypothesis and alternative hypothesis are given below:

$$H_0^1 : \gamma = 0, \kappa = 0 \quad vs \quad H_1^1 : \gamma \neq 0, \kappa \neq 0\,. \tag{10}$$

Whether γ equals zero or not indicates whether limit up has impact on price delayed discovery effect or not. Similarly, whether k equals zero or not indicates whether limit down has impact on price delayed discovery effect or not.

3.2 Volatility model

When studying price volatility, Chou and Wu (1998) assume that volatility is a constant when there are no limit moves. Although volatility spillover is one of the reasons of volatility clustering and other heteroscedastic characteristics indicated from the financial

asset return sequence, there exists volatility clustering phenomena in some futures markets without price limits. So it is not ok to consider heteroscedastic characteristics as the result of price limits. Therefore we use heteroscedastic Volatility Model in our study.

The popular Volatility Models are GARCH (Engle, 1982) model and Stochastic Volatility model (SV model) (Taylor, 1986). These models can accurately describe the special characteristic of the financial data that has conditional fluctuation depending on time. Comparing to GARCH model, SV model has the following advantages: (1) SV model considers historical data and the newest information, which makes the prediction more acceptable; (2) SV model can easily reflects leverage of different types of information. Although GARCH model can also reach this goal after revising, it introduces more parameters; (3) SV model is more flexible comparing to GARCH model with fewer restrains on parameter estimation and testing hypothesis. In addition, SV model can show the fat tail characteristic of asset return sequence. With the recent development of MCMC computing techniques, parameter estimation of SV model is no longer a problem. Therefore, we use SV model to be futures return volatility model.

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Generally, suppose volatility follows the SV(1) process:

$$\sigma_t = \exp(h_t / 2),$$
$$h_t = \omega + \psi(h_{t-1} - \omega) + \eta_t,$$

$$\eta_t \sim N(0, \tau^2),$$

Where $\psi \in (-1, 1)$, η_t and ε_t are mutually independent. Stochastic volatility model considers the logarithm of volatility as an autoregressive process, which is time-dependent. Autoregressive coefficient ψ shows sustained status of fluctuation. η_t Denotes the potential information from trading day t-1 to trading day t. In order to investigate the

impact on futures price volatility due to price limits, we introduce a dummy variable that represents whether there is a limit move or not. The above model becomes:

$$h_t = \omega + \psi(h_{t-1} - \omega) + \zeta\delta_{t-1}{}^{u} + \lambda\delta_{t-1}{}^{d} + \eta_t. \tag{11}$$

The coefficients of dummy variables show the influence of price limits on volatility. ζ >0（λ>0) indicates that volatility increases because of limit move, resulting volatility spillover. ζ <0 (λ<0) indicates that volatility decreases after limit move. Otherwise, ζ =0 (λ = 0) indicates no influence on volatility from price limits. The null hypothesis and alternative hypothesis are as the following:

$$H_0{}^2 : \zeta = 0, \lambda = 0 \qquad vs \qquad H_1{}^2 : \zeta \neq 0, \lambda \neq 0 \tag{12}$$

If $H_0{}^2$ is rejected, we conclude that ς and differ from 0 significantly and there exists volatility spillover effect.

3.3 Parameter estimation and testing methods

Parameter estimation is the major problem that restricts the application of SV model. Methods such as least square estimation, maximum likelihood estimation and other methods do not work. Current popular methods for parameter estimation of SV model include pseudo likelihood method, generalized moment method and MCMC sampling method. All of these methods require a lot of computation. MCMC sampling method is one of the efficient ways to estimate the parameters of SV model. The basic idea of MCMC is from Bayes method. MCMC sampling method considers the parameters as random variables, assigns parameters certain prior distributions and calculates their posterior distributions. Random numbers are drawn from conditional posterior distribution to estimate the parameters.

Let ξ be a vector of random variables with $(u,\varphi,\gamma,\kappa,\omega,\psi,\zeta,\lambda,\tau)$. We set up the prior distribution as the following non-informative improper prior:

$$\pi_0(u,\varphi,\gamma,k,\omega,\psi,\zeta,\lambda,\tau) \propto 1,$$

where

$$u \in (-\infty,+\infty), \varphi \in (-1,1), \gamma \in (-\infty,+\infty), k \in (-\infty,+\infty)$$
$$\omega \in (-\infty,+\infty), \psi \in (-1,1), \zeta \in (-\infty,+\infty), \lambda \in (-\infty,+\infty), \tau \in (0,\infty).$$

To ensure model stability of (9) and (11), we restrict φ and ψ between –1 and 1.

Let $\boldsymbol{r}$ be $\{r_t,\ t = 1,\ldots,T\}$, ξ be the return vector and $\boldsymbol{h} = \{h_t,\ t = 1,\ldots,T\}$ be the log volatility vector. Based on (9), we can get the likelihood function of the return sequence $\{r_t\}$ as the following:

$$L(\xi,\boldsymbol{h}\,|\,\boldsymbol{r}) = \prod_{t=1}^{T} p(r_t\,|\,r_{t-1},h_t,\xi)\,p(h_t\,|\,h_{t-1},\xi)$$

Under the condition that the prior distribution is non-informative, the joint posterior distribution function of the parameters and $\boldsymbol{h}$ is

$$\pi(\xi,\boldsymbol{h}\,|\,\boldsymbol{r}) = L(\xi,\boldsymbol{h}\,|\,\boldsymbol{r}) / \int L(\xi,\boldsymbol{h}\,|\,\boldsymbol{r})\,d\boldsymbol{r} \tag{13}$$

Therefore, Bayes estimator of the parameters are given by:

$$\hat{\xi} = \iint \xi\pi(\xi,\boldsymbol{h}\,|\,\boldsymbol{r})\,d\boldsymbol{h}\,d\xi$$

Notice the dimension of the vector $\boldsymbol{h}$ is the same as the number of the observed data, calculating the above integral directly is impossible. MCMC sampling method (Berger, (1985) is a recently developed efficient Bayes computational method. Its basic idea is to obtain a sample $\pi(\xi,\boldsymbol{h}|\boldsymbol{r})$ by constructing a Markov Chain with stationary distribution

$\pi(\xi,\boldsymbol{h}|\boldsymbol{r})$. Various statistical inferences are then derived based on these samples and large number theory.

The most popular MCMC sampling method is Gibbs sampling. Let $\boldsymbol{\theta}=(\mu,\varphi,\kappa,\gamma,\omega,\psi,\zeta,,\lambda,\tau,\boldsymbol{h})$, following is the basic steps:

Step1: Calculate the stationary distribution $\pi(\boldsymbol{\theta_i}|\boldsymbol{\theta}_{\neq i},\boldsymbol{r})$

$$\pi(\boldsymbol{\theta}\mid \boldsymbol{r}) \propto \pi(\boldsymbol{\theta_i}\mid \boldsymbol{\theta}_{\neq i,}\boldsymbol{r})\ \pi(\boldsymbol{\theta}_{\neq i}|\boldsymbol{r})$$

Where $\boldsymbol{\theta}_{\neq i}$ denotes the vector without $\boldsymbol{\theta}_i$, it is easy to write the posterior distribution (14) as the above form.

Step 2: Choose a starting point $\theta^{(0)}$ such that $\theta^{(0)}=(\theta_1^{(0)},\cdots\cdots,\theta_k^{(0)})$, where k is the length of vector θ;

Step3: Draw $\theta_i^{(j)}$ from stationary distribution $\pi(\theta_i|\theta_{\neq i}^{(j-1)},\boldsymbol{r})$（$i$=1,2,⋯,$k$）;

Step4: Repeat step3, sampling N times after convergence;

Step5: Use the N samples after convergence to perform statistical inference.

Based on the sample obtained by MCMC method, we can estimate the parameters of model (9) and (11) and perform hypothesis testing and other statistical inference. In the later empirical study, besides estimating the parameters, we also calculate the highest posterior density (HPD) interval of the coefficients of dummy variables at certain significance level. In terms of HPD interval (Berger, 1985), we use observed data and the resulted posterior distribution to find a minimum interval within the domain of parameters such that the probability of the parameters in the interval reaches the required confidence level. If the HPD interval includes null hypotheses H_0^1 and H_0^2, we have no reason to reject H_0^1 and H_0^2.

4 Empirical studies

4.1 Data

Currently there are three futures products: copper, aluminum and natural rubber futures in SHFE. Aluminum futures price is quite stable with seldom limits hit. Therefore copper and natural rubber is used in our study. Data we use for study consists daily closing price and settlement price during three-month contract. Table 1 gives the information of the dataset. During the period (1999-2003), each product experienced from recession to market bubble. Therefore, data is representative.

In practice, when judging whether there is a limit hit or not, we do not strictly follow the described percentage, but consider the minimum tick size of the contracts. If we use Δ to denote the minimum tick size and use SP_t to represent settlement price of tth trading day, the price limits of $t+1^{th}$ trading day is:

$$P_{t+1}{}^{u} = \Delta \times \text{int}\left(\frac{SP_t(1+l)}{\Delta}\right), P_{t+1}^{d} = \Delta \times \text{ceil}\left(\frac{SP_t(1-l)}{\Delta}\right) \quad , \tag{14}$$

where function int (x) is the largest integer not greater than x and ceil (x) is the smallest integer not less than x. If the price is higher than $P_{t+1}{}^{u}$, a limit up happens. If the price is lower than $P_{t+1}{}^{d}$, a limit down happens. This rule is slightly different from the rule of the futures Exchange.

Table 1: Description of data

	Beginning and	Tradin	Minimum	Days	Days	Frequen

	ending date	g days	volatility unit	of up limits hit	of lower limits hit	cy of limits hit
Copper futures	1999/2/23—2003/11/20	1096	10 RMB	13	9	2.02%
Natural rubber futures	1999/5/19—2003/11/20	1029	5 RMB	34	28	6.03%

Table 1 gives the beginning and ending dates of the data in our study, number of trading days and information on limit hits of both futures products.

From table 1, we notice that frequency of limits hit is quite different between the two futures products, indicating the difference of fluctuation and risk between these two products. The high frequency of limits hit of natural rubber indicates high price volatility and high risk of this product. The relatively lower frequency of limits hit of copper futures indicates relative stability and relative lower risk of this product.

4.2 Empirical results and analysis

We generate MCMC samples for the parameters of model (9) and (11) using BUGS and SAS software. The result indicates that the sequence converges around 4000 times sampling (Figure 3). We then continue with another 16000 sampling. The converged sampling sequence is used to estimate the parameters. Figure 4 gives the distributions of sampling sequences of dummy variables after convergence, which are asymptotically symmetric. So we can determine 90% HPD confidence interval by calculating its 5% and 95% quartiles.

Table 2 gives the estimator, standard deviation, MCMC sampling error and HPD confidence interval of each parameter. Table 3 gives a summary of the effects of price limits on copper and natural rubber.

4.2.1 Price delayed discovery effect

For copper futures, although both the estimated coefficients of dummy variables γ and k in the means model are not equal to zero, both of the 90% HPD confidence intervals include zero, so we don't reject the null hypotheses $H_0^1 : \gamma = 0, \kappa = 0$. Price limits didn't have significant price delayed discovery effect on copper futures. Coefficient γ is greater than zero, indicating slight price delayed discovery effect. Coefficient k is less than zero, indicating that price will be pulled back in the following trading days. Overreaction exists in copper futures markets, which is corrected in the following trading days.

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For natural rubber futures, the estimated coefficient of γ in the means model is not equal to zero and zero is outside its 90% HPD confidence interval. The hypothesis $\gamma = 0$ is therefore rejected. There is significant price delayed discovery effect on natural rubber futures due to limit up. Coefficient *k* is less than zero, but zero is inside its 90% HPD confidence interval, indicating slight delayed discovery effect of natural rubber futures price after limit down.

4.2.2 Volatility spillover effect

For copper futures and natural rubber volatility model, all the 90% HPD confidence intervals of dummy variables include zero. We can't reject hypothesis H_0^2 for both products. This indicates that no significant volatility spillover effect on copper futures and

natural rubber futures is due to price limits. Both of the dummy variables of limit up are not equal to zero, indicating slight volatility spillover effect on copper futures and natural rubber futures because of limit up. Coefficients of limit down of copper futures and natural rubber futures are less than zero and greater than zero respectively, indicating that there is slight decrease of copper futures return volatility following limit down. This means that price limits have cooling-effect on the copper investors. While natural rubber futures return volatility has slight spillover phenomena.

From the above analysis, price limits have no adverse impact on copper futures price. It is efficient to control risks for copper futures. For natural rubber, limit down has no adverse impact, while the limit up hinder futures price-discovery process.

The difference between the effects of price limits on natural rubber futures and copper futures is related to the different markets behavior of these two products. Copper futures are relatively stable with few limits hit. The frequency of limits hit is only 2.02%. While the frequency of limits hit of natural rubber futures is 6.03%. Price limits have more influence on active futures products than the relatively stable ones.

When studying the effect of price limits on stock price, Kim and Limpaphayom (2000) pointed out those different effects from price limits existed because of different characteristics of stocks. Based on our study, their conclusions can be extended to futures markets.

Table 2: The results of MCMC estimation

Parameter	Mean	Standard deviation	Sampling error	90%HPD interval

Copper				
$\mu(10^{-2})$	-0.0046	0.0214	0.0007	
$\varphi(10^{-2})$	-9.2380	3.4450	0.1172	
$\gamma(10^{-2})$	0.5560	0.3730	0.0086	(-0.0512,1.1720)
$k(10^{-2})$	-0.2478	0.3879	0.0077	(-0.8778, 0.3810)
ω	-9.7370	0.1964	0.0085	
Ψ	0.9379	0.0262	0.0023	
ζ	0.1317	0.2351	0.0195	(-0.2464, 0.5244)
λ	-0.3547	0.2999	0.0232	(-0.8462, 0.1385)
τ	0.2646	0.0537	0.0052	
Natural rubber				
$u(10^{-2})$	0.0014	0.0241	0.0008	
$\varphi(10^{-2})$	-8.1730	3.5620	0.1004	
$\gamma(10^{-2})$	1.0800	0.3350	0.0099	(0.5322,1.6370)
$k(10^{-2})$	-0.6022	0.4060	0.0136	(-1.2700, 0.0599)
ω	-9.4560	0.2010	0.0076	
ψ	0.9039	0.0281	0.0022	
ζ	0.1633	0.1843	0.0118	(-0.1228, 0.4826)
λ	0.1931	0.2168	0.0145	(-0.1441, 0.5657)
τ	0.4753	0.0590	0.0054	

Table 2 gives the estimator, standard deviation, MCMC sampling error of each parameter and HPD confidence interval of the dummy variables.

Table 3: A summary of the effect of price limits on copper and natural rubber futures

	Upper Limit		Lower Limit	
	Price Discovery	Volatility	Price Discovery	Volatility
Copper	Slightly delay	Slightly spillover	Slightly pulling back	Slightly decreasing

Natural rubber	Significantly delay	Slightly spillover	Slightly pulling back	Slightly spillover

Table 3 gives a summary of the effect of price limits on copper and natural rubber. Price limits have no adverse impact on copper futures price. It is efficient to control risks for copper futures. For natural rubber, limit down has no adverse impact, while the limit up hinder futures price-discovery process.

5. Conclusions

This paper studies the effect of price limits on SHFE empirically and theoretically using autoregressive model, stochastic volatility model and MCMC sampling techniques. The authors have the following findings: (1) Price limits have no adverse impact on copper futures price. They are efficient to control risks. For natural rubber futures, although price limits efficiently control risks, it impede futures price discovery process. (2) The influence on futures price from price limits relies on characteristics of futures products. Influence is significant for products with large fluctuation and frequent limits hit.

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Appendix: Sampling sequences and sampling distributions of the coefficients of dummy variables for Natural Rubber Futures.

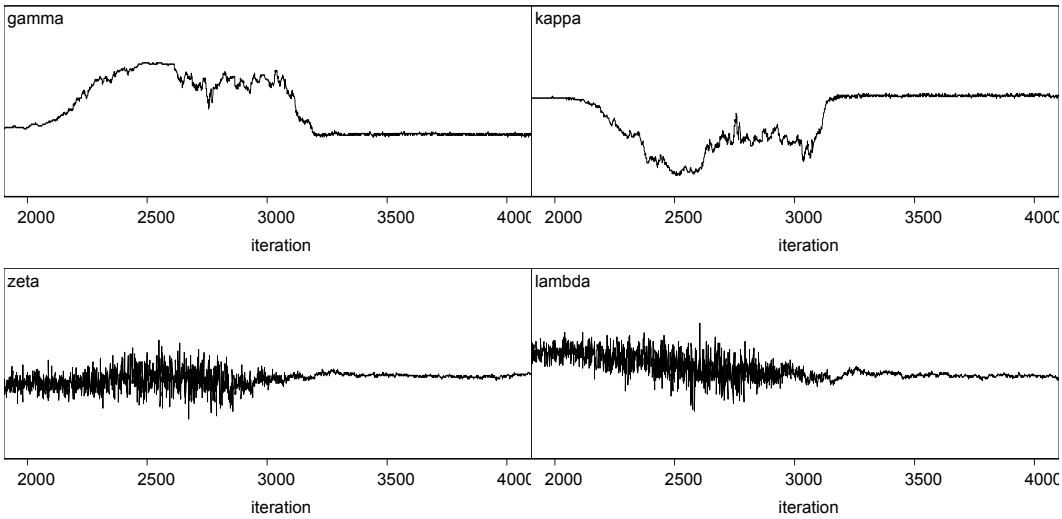

Figure 3

Figure 3 gives a description of the convergence of the sampling sequences of the coefficients of dummy variables for Natural Rubber Futures. The sequences converge around 3100 sampling.

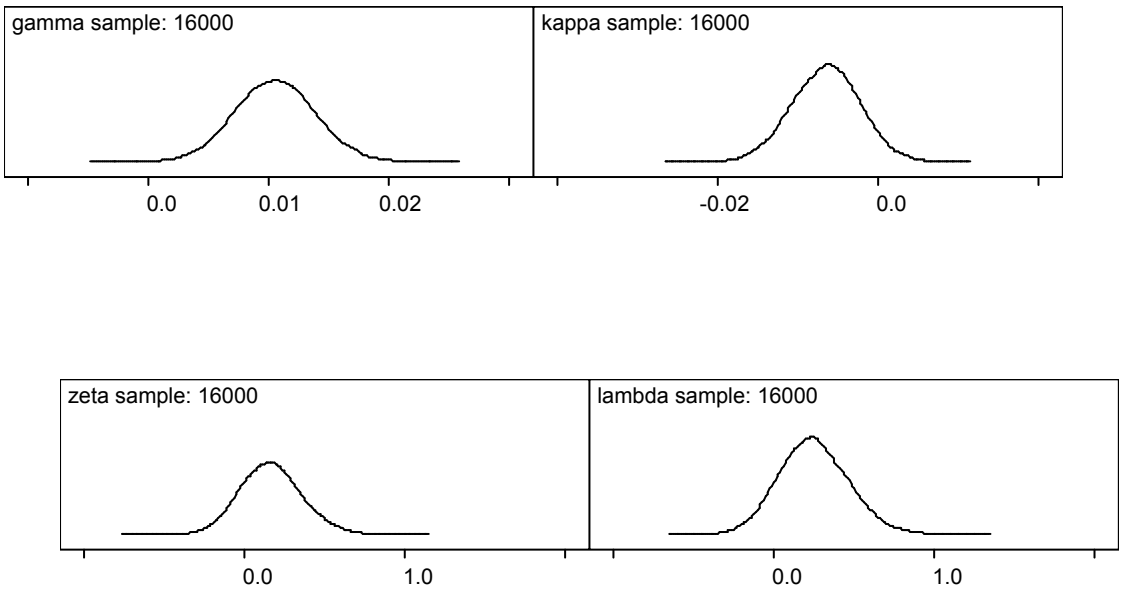

Figure 4

Figure 4 gives the sampling Distribution of Dummy Variables' Coefficients for Natural Rubber Futures. All the distributions are asymptotically symmetric.