

A new finite depth subfactor which appears in a quadrilateral of factors  
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A quadrilateral of factors is a pair of intermediate subfactors  $N \subset P \subset M$ ,  $N \subset Q \subset M$  s.t.  $P \vee Q = M$ ,  $P \wedge Q = N$ . We show that there is a quadrilateral of factors with some conditions, so that  $P \subset M$  is a subfactor with index  $(5 + \sqrt{17})/2$ , and  $N \subset P$  is a subfactor with index  $(7 + \sqrt{17})/2$ . This work is joint with Pinhas Grossman.

Generators of some non-commutative stochastic processes

Michael Anshelevich  
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The following results are well known. A Levy process is a Markov process. Its transition operators form a semigroup of contractions. The generator of this semigroup can be written down explicitly using Fourier analysis.

I will discuss the same questions in the framework of free Levy processes (process with freely independent, stationary increments). Some of the new difficulties in this case are (1) there is no free Fourier transform, and (2) the transition operators no longer form a semigroup. Nevertheless, most of the results from the first paragraph carry over, with the generators written down explicitly as singular integral operators.

All the free probability results needed will be quoted but not proved, so only functional analysis is necessary as background.

On the symmetric enveloping algebra of planar algebra subfactors

Stephen Curran  
University of California, Los Angeles

In recent work, Guionnet, Jones and Shlyakhtenko gave a purely diagrammatic construction of a subfactor from a planar algebra. In this talk we will describe a diagrammatic construction of Popa's symmetric enveloping algebra for these subfactors. We will discuss how this description can be used to compute some invariants of Ocneanu's asymptotic inclusion, which were previously computed by Ocneanu and Evans-Kawahigashi using TQFT methods.

This talk is based on joint work with V. Jones and D. Shlyakhtenko.

On Mathematical and Physical Applications of Matrix Convexity

Edward G. Effros  
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As one might expect from classical functional analysis, the notion of matrix convexity underlies the foundations of quantized functional analysis. In 1996, Soren Winkler and I proved an analogue of the geometric Hahn-Banach theorem. This had several important applications. On the one hand it enabled Webster and Winkler to find an exact analogue of Kadison's theorem that any compact convex set in a locally convex space can be realized as the *state space* of a suitable ordered linear space. They used the result to prove a striking

analogue of the Krein-Milman theorem. But even more intriguing is that Winkler succeeded in formulating a quantum version of the Legendre transform.

As is well-known, a characteristic property of both classical and quantum thermodynamics is that the fundamental variables are convex, but not necessarily smooth functions. In fact, phase transitions are reflected in the differential singularities of such functions.

The convexity properties of quantum variables is quite subtle. For example the joint convexity of relative entropy remained an open problem for a period. It is now understood that once one acknowledges the underlying matrix convexity, the proof is elementary.

It is the Legendre transform that is more puzzling. From the classical point of view, the graph of a non-pathological convex function is the envelope of its tangent lines. The Legendre transform describes the parametrized family of lines. In the quantum context, this is not fully understood, since the existence of supporting affine spaces is no longer guaranteed. The problem is none-the-less fascinating, since it seems related to a notion of “matrix derivative”. Hopefully, the singularities, i.e., phase transitions of non-smooth variables might display a finer structure which is encoded in the matrix convexity framework.

If there is time, I will also briefly touch on Helton and McCulloch’s surprising application of the EW result to classical control theory.

*K*-theoretic classification of free-fermion Hamiltonians  
Alexei Kitaev  
California Institute of Technology

I consider Hermitian (and other types of) matrices whose rows and columns are associated with points in some compact space  $Y$  (assumed to be a nice subset of  $\mathbb{R}^n$ ), and nonzero elements are only allowed between nearby points. The matrix spectrum is bounded away from zero (a local variant of this condition is actually used). If we take such matrices up to continuous deformation and pass to the Grothendieck group, their equivalence classes are classified by  $K$ -homology of the space  $Y$ . The proof is based on a coarse-graining procedure (a  $K$ -theoretic analogue of convolution), which results in a Dirac-like operator with a mass term. The role of the convolution kernel is played by the Jackiw-Rebbi soliton.

Hyperbolic groupoids and their  $C^*$ -algebras  
Volodymyr Nekrashevych  
Texas A&M University

The general notion of a hyperbolic groupoids includes as partial cases actions of Gromov hyperbolic groups on their boundaries, groupoids generated by expanding maps, Ruelle groupoids of Smale spaces, etc. An interesting aspect of the theory of hyperbolic groupoids is a duality theorem: for every hyperbolic groupoid  $G$  there is a naturally defined groupoid  $G^\top$  acting on the boundary of  $G$ , and  $(G^\top)^\top$  is equivalent to  $G$ . We will discuss properties of the  $C^*$ -algebras associated with hyperbolic groupoids, the corresponding duality theory, and applications to dynamics.

Fractal wavelets and frames and related operators on solenoid spaces  
Judith A. Packer

University of Colorado, Boulder

We discuss a construction, first due to D. Dutkay and P. Jorgensen, that is used to define generalized wavelets on inflated fractal spaces arising from iterated function systems. Self-similarity relations defining the fractal spaces also give rise to filter functions defined on the torus. These filter functions can be used to construct isometries as well as probability measures on solenoids. Representations of the Baumslag-Solitar group can be obtained from the probability measures, and properties of the representation are related to properties of the original wavelet and filter systems. In addition to wavelets, we discuss semi-orthogonal frames on fractal spaces first constructed by J. D'Andrea. This work is joint with L. Baggett, K. Merrill, and A. Ramsay.

On the classification of non-simple graph  $C^*$ -algebras

Efren Ruiz

University of Hawaii at Hilo

We discuss the classification problem of non-simple graph  $C^*$ -algebras. In particular, we show how  $K$ -theory together with its scale can be used to classify graph  $C^*$ -algebras with exactly one non-trivial ideal. Time permitting, we will also discuss the question of determining when extensions of two graph  $C^*$ -algebras is again a graph  $C^*$ -algebra.

Is there a mod Hilbert-Schmidt BDF type theorem for operators with trace-class selfcommutator?

Dan Voiculescu

University of California at Berkeley

We revisit the question in the title. We show that part of the problem is about the  $K$ -theory of certain Banach algebras.

Nonstandard  $q$ -deformations

Hans Wenzl

University of California, San Diego

We construct finite depth subfactors using a second  $q$ -deformation of Brauer's centralizer algebras, which describes the decomposition of tensor powers of the vector representation of an orthogonal group. In the classical limit  $q \rightarrow 1$  they would correspond to the infinite index fixed point subfactors  $R^{U(N)} \subset R^{O(N)}$ , for suitable actions of the groups  $O(N) \subset U(N)$  on the hyperfinite subfactor  $R$ . These subfactors can be linked to a nonstandard  $q$ -deformation  $U'_q so_N \subset U_q sl_N$ ; the usual Drinfeld-Jimbo  $q$ -deformation  $U_q so_N$  does not allow such an embedding. A general understanding of such  $q$ -deformations is expected to give a general construction of subfactors for a large class of symmetric spaces.