I forgot this in lecture.

Example 1. Let S denote the set of all 3-by-3 matrices that are either upper or lower triangular. Is S a subspace?

Certainly S has lots in it, like

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \in S$$

and also

$$B = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix} \in S.$$

However, if we add these, we get

$$A + B = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 3 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

that is not upper triangular and not lower triangular. It is not is S. We just found "vectors" A and B so that

$$A \in S \text{ and } b \in S \text{ and yet } A + B \notin S.$$

So one of the closure properties fails, at least in one place. That's it. **We conclude** S is not a subspace.