**Problem 1.** Define $L$ from $\mathbb{R}^{2 \times 2}$ to $\mathbb{R}^{2 \times 2}$ by

$$L(A) = A^T.$$ 

Since $A^T + B^T = (A + B)^T$ and $(\alpha A)^T = \alpha A^T$ this is a linear transformation.

(a) With respect to the ordered basis

$$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

what is the matrix representing $T$? (This means “with respect to $B$ and $B$,” so the same ordered basis is at work in the domain and target.)

(b) What is $L(\mathbb{R}^{2 \times 2})$?

(c) What is $\ker(L)$?

**Problem 2.** Suppose $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by

$$T \left( \begin{bmatrix} a \\ b \end{bmatrix} \right) = \begin{bmatrix} 2a \\ a + b \end{bmatrix}.$$ 

What is the matrix representation of $T$ with respect to

$$B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}?$$

**Problem 3.** Suppose $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ has matrix representation with respect to

$$B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

equal to

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$ 

How is $T$ defined?

**Problem 4.** Problem 14, page 183, section 4.1.
**Problem 5.** Determine if $L$ is a linear transformation from $C[0, 1]$ to $\mathbb{R}$ if

$$L(f) = \frac{1}{3} f(0) + \frac{2}{3} f(1).$$

(See problem 11, but you need to give reasoning, not just a yes or no answer like the back of the book.)

**Problem 6.** Determine if $L$ is a linear transformation from $C[0, 1]$ to $\mathbb{R}$ if

$$L(f) = (f(0))^2.$$  

**Problem 7.** Problem 13, page 206, section 4.3.