HOMEWORK 6

Problem 1. In $\mathbb{R}^3$ there are vectors $\mathbf{v}_1$,$\mathbf{v}_2$ and $\mathbf{v}_3$ and suppose you have suceeded in finding that

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 2\mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3,$$

showing that this vector is in the span of the $\mathbf{v}_j$. You friend got the answer

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \mathbf{v}_1 + 2\mathbf{v}_2 + \mathbf{v}_3.$$

Is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ a linearly dependent set? If so, what linear combination of these three vectors equals zero, besides the zero combination?

Problem 2. Apply the basis casting out technique (read the handout) to these "vectors".

$$A_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

You should be able to solve in your head the linear combination equations that arise.

Problem 3. Apply the accelerated casting out to these vectors in $\mathbb{R}^5$. (See hand out, the "Fast, but only in..." section). Do the row operations by hand, but *stop when you see the pivots (at weak row echelon form.)*

$$\begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}$$

Problem 4. Let

$$\mathbf{b}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Now consider the ordered basis $[\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$ of the subspace of 5-vectors that are symmetric with repect to a vertical flip.

With respect to this ordered basis:

(a) What is the coordinate vector of

$$\begin{bmatrix} 5 \\ 4 \\ 3 \\ 4 \\ 5 \end{bmatrix}?$$

(b) What is the coordinate vector of

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}?$$

(c) Which 5-vector has coordinate vector

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}?$$

(d) Which 5-vector has coordinate vector

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}?$$

Problem 5. In the 2-by-2 matrices, consider

$$A_1 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

With respect to the ordered basis $[A_1, A_2, \ldots, A_4]$ answer:

(a) What is the coordiate vector of

$$C = \begin{bmatrix} 4 & 8 \\ 0 & 2 \end{bmatrix}?$$

(b) What is the coordiate vector of

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}?$$

(c) What is E if it has coordinate vector

$$\begin{bmatrix} 10 \\ 5 \\ 5 \\ -5 \end{bmatrix}$$

Problem 6. In the 2-by-2 matrices, consider

$$F_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \quad F_3 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad F_4 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

With respect to the ordered basis $[F_1, F_2, \ldots, F_4]$ answer the following. You may use the `rref` function of Matlab to solve any systems that arise. Just say "Matlab tells me $X \sim Y$."

(a) What is the coordiate vector of

$$C = \begin{bmatrix} 4 & 8 \\ 0 & 2 \end{bmatrix}?$$

(b) What is the coordiate vector of

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}?$$

(c) What is E if it has coordinate vector

$$\begin{bmatrix} 10 \\ 5 \\ 5 \\ -5 \end{bmatrix}$$