HOMEWORK

Problem 1. Let

\[ A = \begin{bmatrix} 1 & -2 & -3 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 1 & -2 & -3 & 0 & 0 & 2 \end{bmatrix}. \]

(a) Find a basis for the null space of \( A \).
(b) Find a basis for the column space of \( A \).
(c) What is the rank of \( A \)?
(d) What is the nullity of \( A^T \)?

Problem 2. Let

\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -2 \\ 0 & -4 & 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 & -1 & 0 & 2 \end{bmatrix}. \]

(a) Find a basis for the row space of \( A \).
(b) Find a basis for the column space of \( A \).

Problem 3. Consider the set of vectors comprised of

\[ v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 4 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ 5 \\ -1 \\ 6 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 6 \end{bmatrix}. \]

Find a subset of this set of vectors that is a basis for their span.

Problem 4. Number 8 in §4.1.
**Problem 5.** Consider $V$ with ordered basis

$$\mathcal{B} = [b_1, b_2, b_3]$$

and $W$ with ordered basis

$$\mathcal{C} = [c_1, c_2, c_3]$$

Suppose $T$ is a linear transformation from $V$ to $W$. Suppose

$$T(b_1) = c_1 + 2c_2$$

and

$$T(b_1 + b_2) = c_1 + 3c_3$$

and

$$T(b_1 + b_2 + b_3) = 3c_1 + 4c_2.$$  

(a) What is the matrix representing $T$ with respect the the bases $\mathcal{B}$ and $\mathcal{C}$? 
(b) What is the matrix representing $T$ with respect the the bases 

$$[b_1, b_1 + b_2, b_1 + b_2 + b_3]$$

and $\mathcal{C}$?  

(Correction: Added $c_3$ to $\mathcal{C}$.)

**Problem 6.** Consider the linear transformation $T$ from $\mathbb{R}^3$ to $\mathbb{R}^4$ defined by

$$T \left( \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = \begin{bmatrix} a \\ b \\ c \\ a \end{bmatrix}.$$  

(a) What matrix $A$ satisfies 

$$T(v) = Av$$

for all $v$ in $\mathbb{R}^3$? 
(b) With respect to the ordered basis

$$b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

and the ordered basis

$$c_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad c_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad c_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad c_4 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix},$$

what is the matrix representation of $T$?  

(Corrections: dropped extra parentheses in equation defining $T$. Change the second $c_3$ into a $c_4$.)