HOMEWORK #8

Problem 1. Consider the following matrix that depends on parameters r and s that can vary of the real numbers:

$$A = \begin{bmatrix} 3r^3 & r^2 & rt \\ 3r^2 & r+t-1 & t+1 \\ 6r^2 & 2r & 2rt+t-2 \end{bmatrix}$$

For which values of the parameters is this invertible?

Problem 2. Let

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find the eigenvalues (with or without finding the eigenvectors) for the following matrices.

(1) A
(2) B
(3) ABA^{-1}
(4) BAB^{-1}
(5) A^T

Problem 3. Use the method of Example 4.29 (in the second edition... this is diagonalizing $A = WDW^{-1}$ and using $A^n = WD^nW^{-1}$....) to compute the following.

What is A^{-8} when

$$A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}?$$

Problem 4. Consider

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 2 & -2 \\ 2 & 1 & -1 \end{bmatrix}.$$

It is hard to find the characteristic polynomial here as there are no zeros and $\det(A - \lambda I)$ has a λ in every row and column. Let me calculate and factor the characteristic polynomial. I start with negating all three rows so the intermediate polynomials look

better.

$$\begin{aligned}
\begin{vmatrix} 3-\lambda & 1 & -2 \\ 2 & 2-\lambda & -2 \\ 2 & 1 & -1-\lambda \end{vmatrix} &= -\begin{vmatrix} \lambda-3 & -1 & 2 \\ -2 & \lambda-2 & 2 \\ -2 & -1 & \lambda+1 \end{vmatrix} \\
&= -\left((\lambda-3)\begin{vmatrix} \lambda-2 & 2 \\ -1 & \lambda+1 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ -2 & \lambda+1 \end{vmatrix} + 2\begin{vmatrix} -2 & \lambda-2 \\ -2 & -1 \end{vmatrix}\right) \\
&= -\left((\lambda-3)\left((\lambda-2)(\lambda+1)+2\right) + (-2(\lambda+1)+4) + 2\left(2+2(\lambda-2)\right)\right) \\
&= -\left((\lambda-3)\left(\lambda^2-\lambda\right) - 2(\lambda-1) + 4(\lambda-1)\right) \\
&= -(\lambda-1)\left((\lambda-3)\lambda-2-4\right) \\
&= -(\lambda-1)\left(\lambda^2-3\lambda-6\right) \\
&= -(\lambda-1)(\lambda-1)(\lambda-2) \\
&= -(\lambda-1)^2(\lambda-2)
\end{aligned}$$

(1) What are the eigenvalues of A, and for each eigenvalue, what are the algebraic and geometric multiplicities?
(2) What are the eigenvalues of A^{-2}, and for each eigenvalue, what are the algebraic and geometric multiplicities?

Problem 5. Suppose A is a square matrix so that

$$A^3 = A^2 + I.$$

Following the argument in exercise 4.3.34 (in the second edition), find formulas for the following. For full credit, give your answers are polynomials in A of degree at most 2.

(1) A^{-1}
(2) A^4
(3) A^5
(4) A^{-2}