HOMEWORK #5

Problem 1. Write out a proof that shows a diagonal matrix that has no zeros on the diagoal has inverse that is also a diagonal matrix, found by inverting each diagonal element.

Problem 2. Suppose

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

Find the following, giving answers that are explicit three-by-three matrices.

(1) A^{-1}
(2) B^{-1}
(3) $B^{-1}AB$
(4) BAB^{-1}
(5) $\left(B^{-1}AB\right)^{-1}$

Problem 3. Find these three inverses using the Gauss-Jordan method. Use only elementary row operations, doing one at a time.

(1)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}^{-1} = ?$$

(2)

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}^{-1} = ?$$

(3)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}^{-1} = ?$$

Problem 4. Suppose $A^4 = 0$ (here 0 is the n-by-n zero).

(1) Show that the inverse of $I - A$ is $I + A + A^2 + A^3$.
(2) What is the inverse of $I + A$?
(3) What is the inverse of $I + 2A$?

Problem 5. Let

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

(1) Find L_1 lower triangular and U_1 upper triangular so that

$$A = U_1 L_1.$$

(2) Find L_2 lower triangular and U_2 upper triangular so that

$$A = L_2 U_2.$$

Problem 6. Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

(1) Find L that is lower triangular, P that is a permutation matrix and U that is upper triangular so that

$$A = LPU.$$

Notice the order is not the one in the book. See the posted examples. This took me five elementary row-op-column-op pairs.

(2) Find L_1 that is lower triangular, P_1 that is a permutation matrix and U_1 that is upper triangular so that

$$A^{-1} = U_1 P_1 L_1.$$

This has an easy answer, based on the answer to the first part.

(3) Find L_2 that is lower triangular, P_2 that is a permutation matrix and U_2 that is upper triangular so that

$$A^{-1} = L_2 P_2 U_2.$$

This took me eight elementary row-op-column-op pairs, and involved fractions and negative numbers. Talk to me if you want a tutorial in Matlab to help checking your work and doing row operations.