Problem 1. Write out a proof that shows a diagonal matrix that has no zeros on the diagonal has inverse that is also a diagonal matrix, found by inverting each diagonal element.

Problem 2. Suppose

\[
A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.
\]

Find the following, giving answers that are explicit three-by-three matrices.

(1) \(A^{-1}\)
(2) \(B^{-1}\)
(3) \(B^{-1}AB\)
(4) \(BAB^{-1}\)
(5) \((B^{-1}AB)^{-1}\)

Problem 3. Find these three inverses using the Gauss-Jordan method. Use only elementary row operations, doing one at a time.

(1)

\[
\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}^{-1} = ?
\]

(2)

\[
\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}^{-1} = ?
\]

(3)

\[
\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}^{-1} = ?
\]

Problem 4. Suppose \(A^4 = 0\) (here 0 is the \(n\)-by-\(n\) zero).

(1) Show that the inverse of \(I - A\) is \(I + A + A^2 + A^3\).
(2) What is the inverse of \(I + A\)?
(3) What is the inverse of \(I + 2A\)?
Problem 5. Let

\[ A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \]

(1) Find \( L_1 \) lower triangular and \( U_1 \) upper triangular so that
\[ A = U_1 L_1. \]

(2) Find \( L_2 \) lower triangular and \( U_2 \) upper triangular so that
\[ A = L_2 U_2. \]

Problem 6. Let

\[ A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 2 \end{bmatrix} \]

(1) Find \( L \) that is lower triangular, \( P \) that is a permutation matrix and \( U \)
that is upper triangular so that
\[ A = LPU. \]

Notice the order is not the one in the book. See the posted examples.
This took me five elementary row-op-column-op pairs.

(2) Find \( L_1 \) that is lower triangular, \( P_1 \) that is a permutation matrix and
\( U_1 \) that is upper triangular so that
\[ A^{-1} = U_1 P_1 L_1. \]

This has an easy answer, based on the answer to the first part.

(3) Find \( L_2 \) that is lower triangular, \( P_2 \) that is a permutation matrix and
\( U_2 \) that is upper triangular so that
\[ A^{-1} = L_2 P_2 U_2. \]

This took me eight elementary row-op-column-op pairs, and involved
fractions and negative numbers. Talk to me if you want a tutorial in
Matlab to help checking your work and doing row operations.