EULER CIRCUITS AND TOURS AND CHINESE POSTMEN

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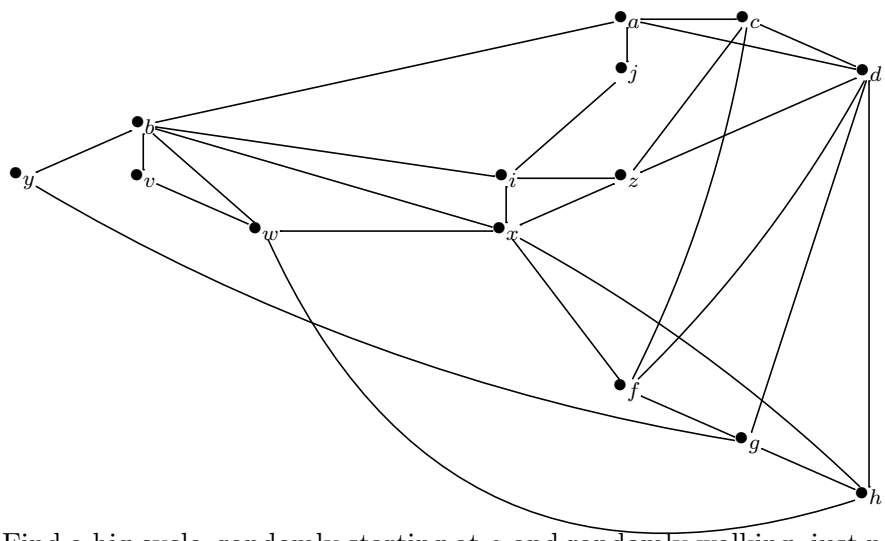
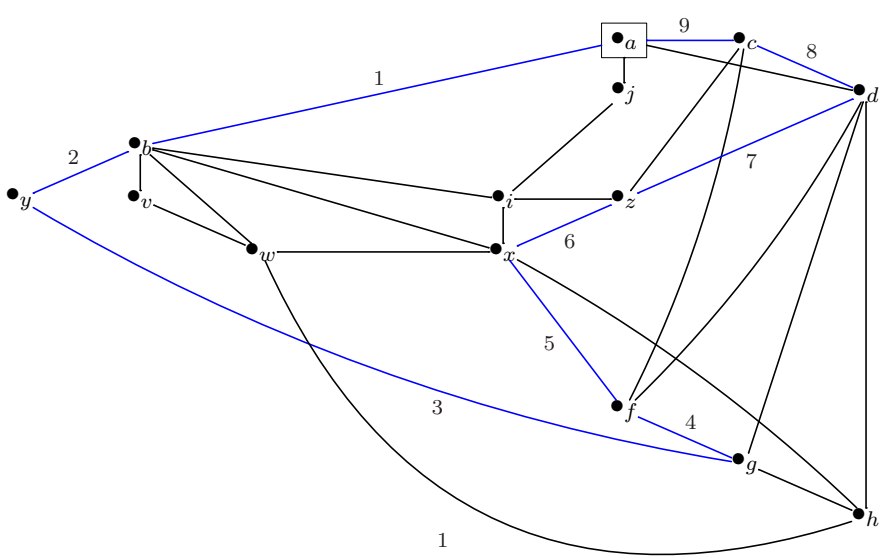
0.1. **Finding Euler Circuits.**

Example 1. Find an Euler circuit for the following graph. Do not use Fleury's algorithm. Instead:

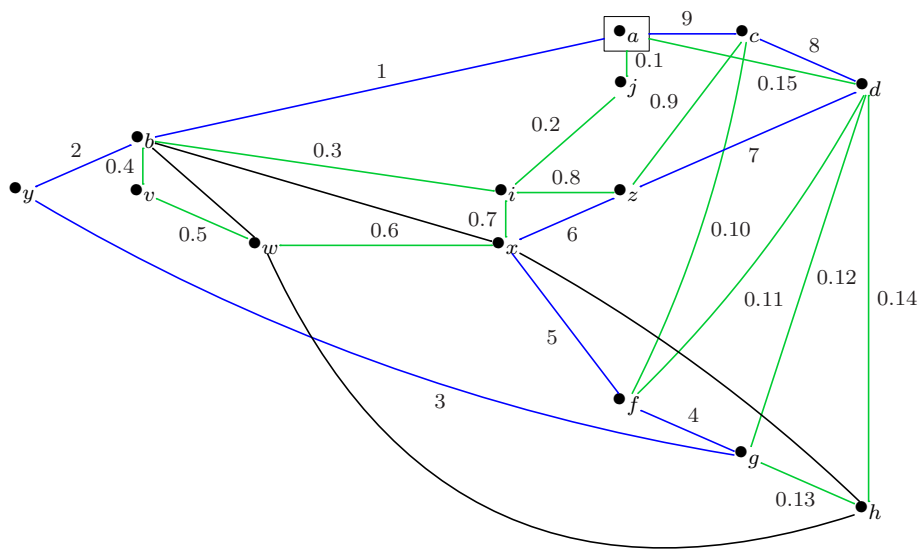
(a) Find a circuit with no repeated edges.
(b) Starting at a vertex within the previous circuit, find a circuit from the unused edges that does not repeat any edges. Splice this into the previous circuit to make a circuit with no repeated edges.
(c) If out of edges, you are done. Else repeat step (a)



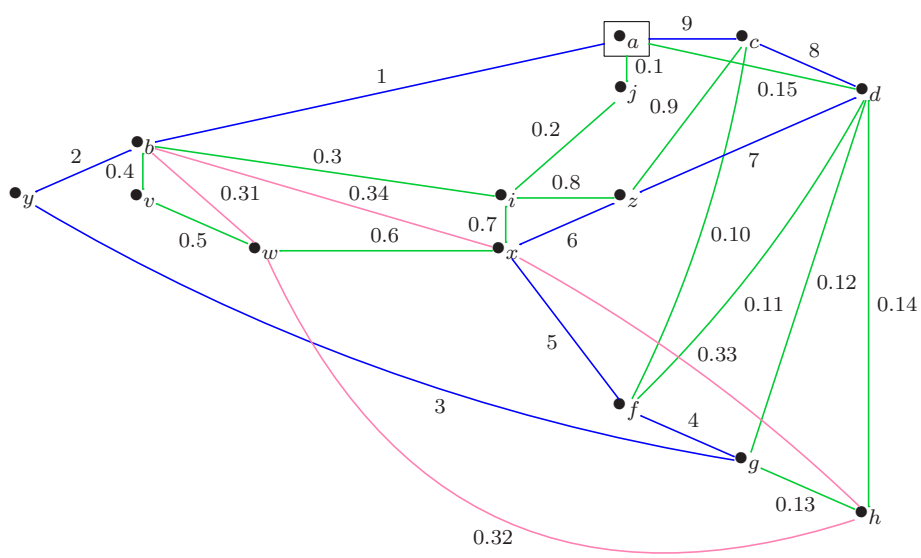
Remark 1. Find a big cycle, randomly starting at a and randomly walking, just not repeating an edge.



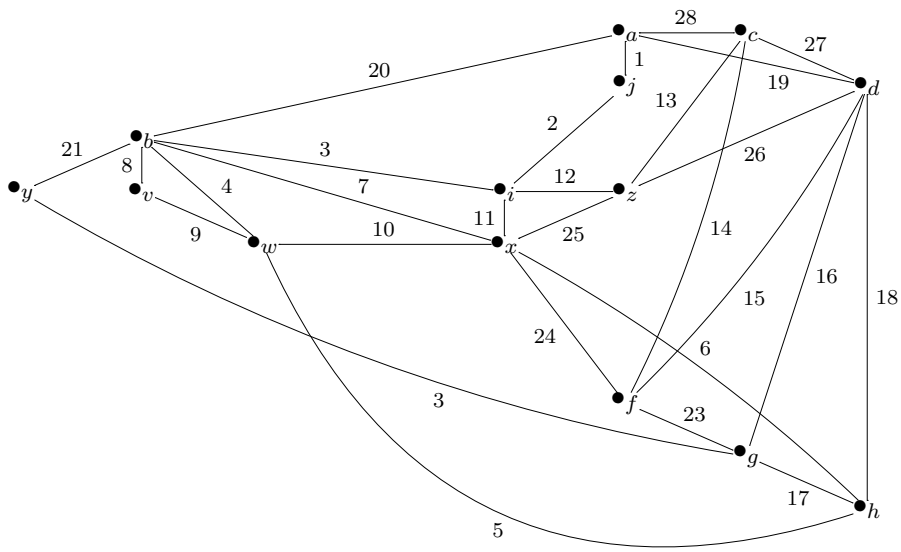
We did not finish that task at a, so let's find another circuit in the black edges starting at a. By numbering its edges with reals between 0 and 1 we are splicing this in at the start.



Now we must add a circuit starting at b. By numbering its edges with reals between 0.3 and 0.4 we are choosing to splice the new, blue cycle in the middle of the green circuit.

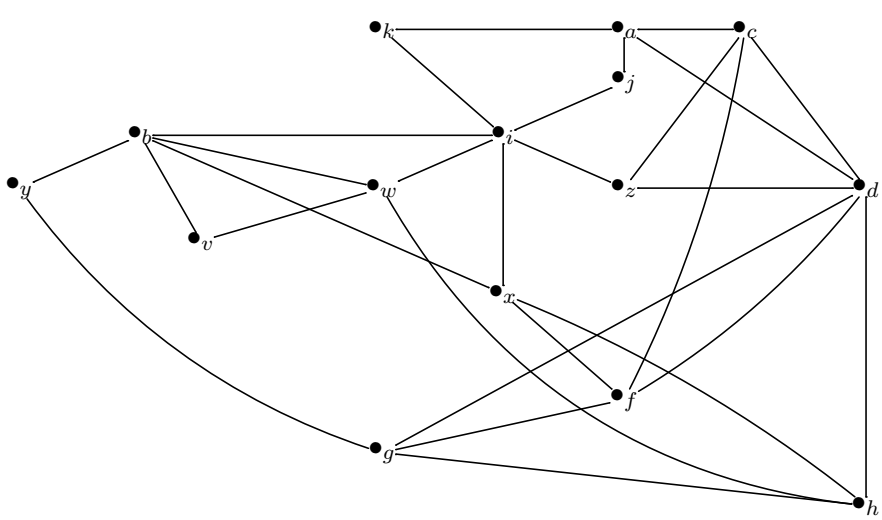


This is a reasonable way to leave the solution; the cycle is the series of all the edges taken in the numerical order. For a pretty answer, you can renumber these $1, 2, \ldots$

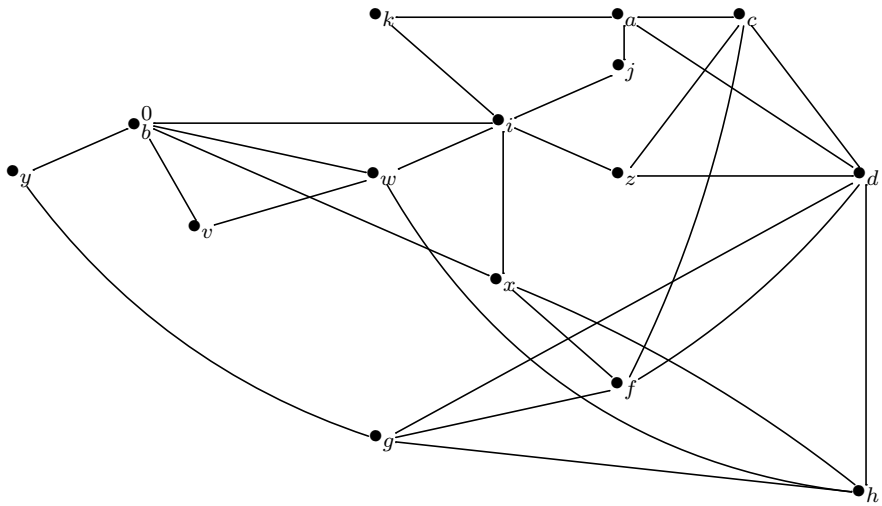


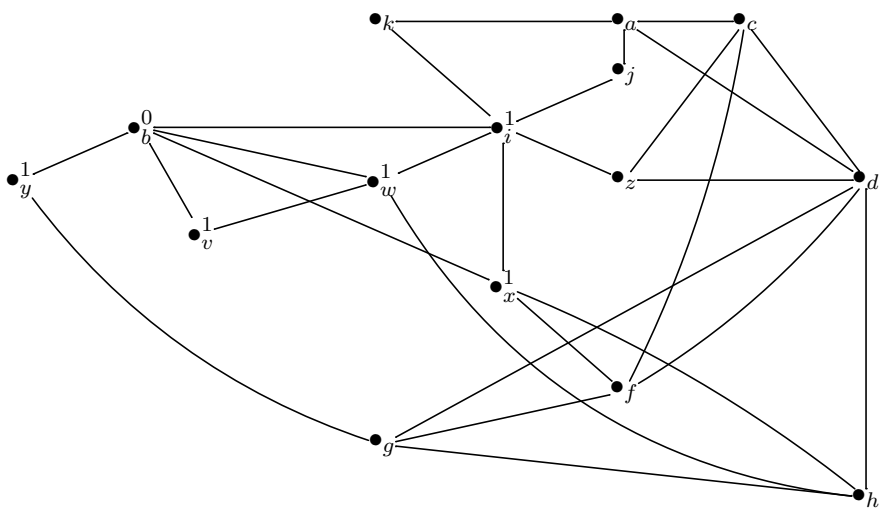
0.2. **The Chinese Postman Problem.** If one has two odd vertices in a connected graph, one can find an Euler Tour. This will start at one odd vertex, hit every edge once, and then end at the other odd vertex. If one wants to insist on a circuit, one needs to repeat some edges. To minimize the number of edges repeated, one needs to find a minimum path between these vertices to be the list of edges repeated twice.

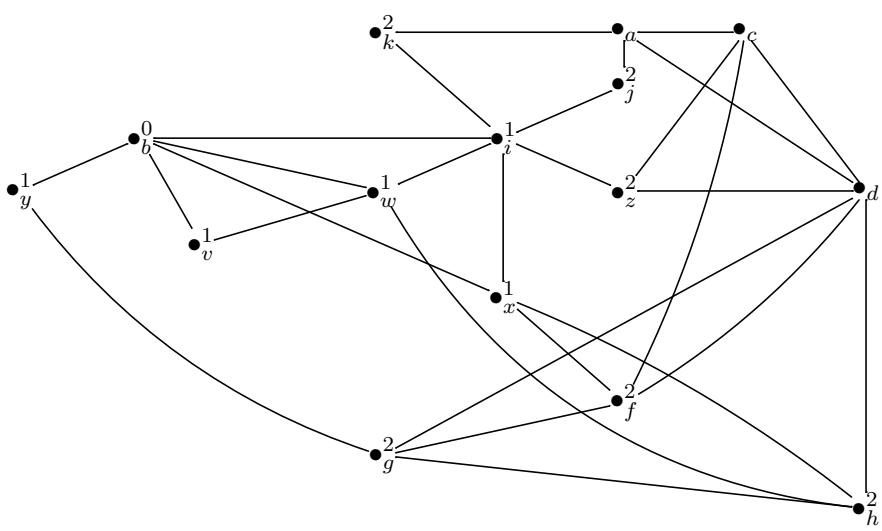
Remark 2. Find a circuit in G that includes every edge at least once and is a short as is possible:



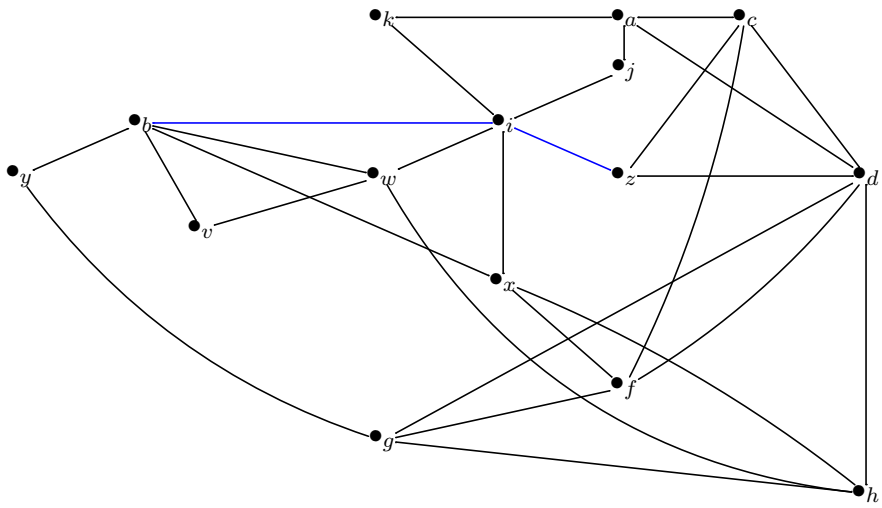
The two odd vertices are b and z. Let us start marking vertices in terms of distance from b :



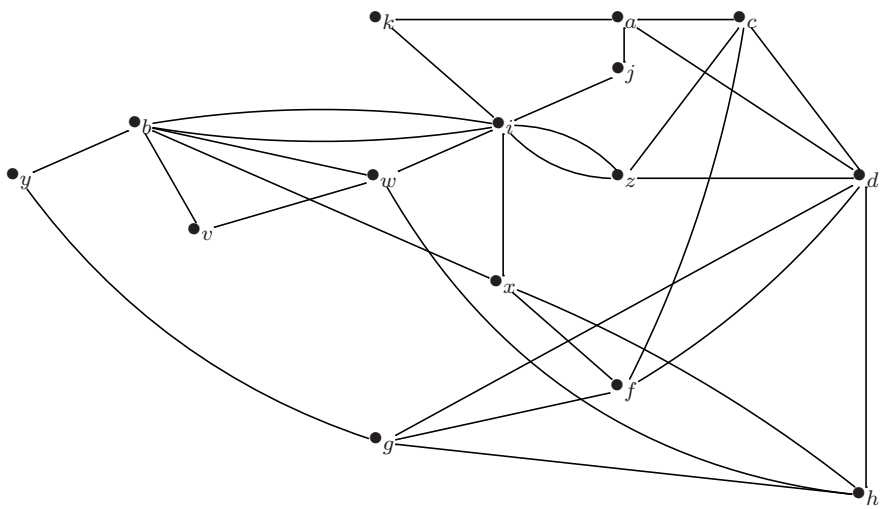




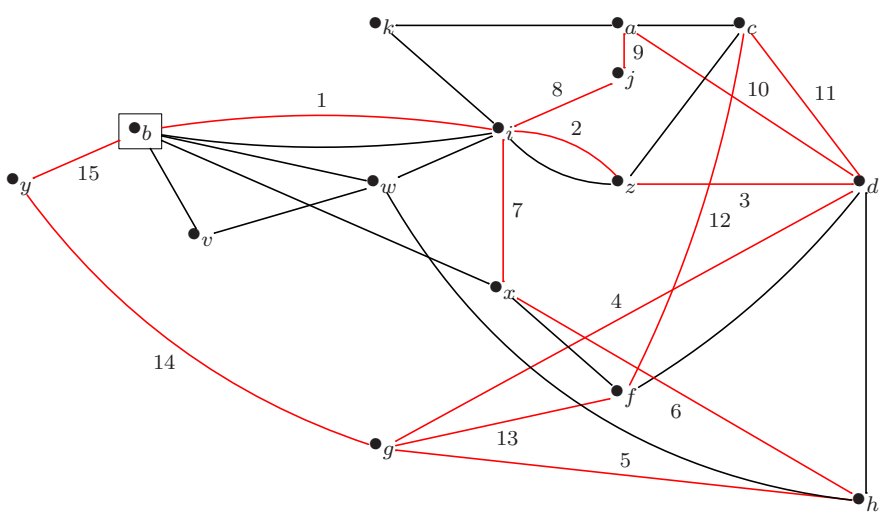
We see the shortest paths from b to z are of length 2, as is, for example, (b, i, z).



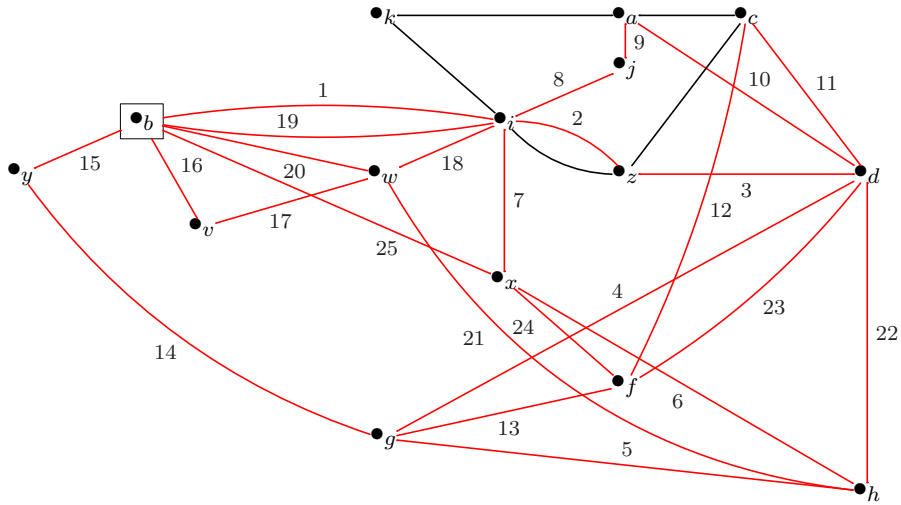
Double those edges:



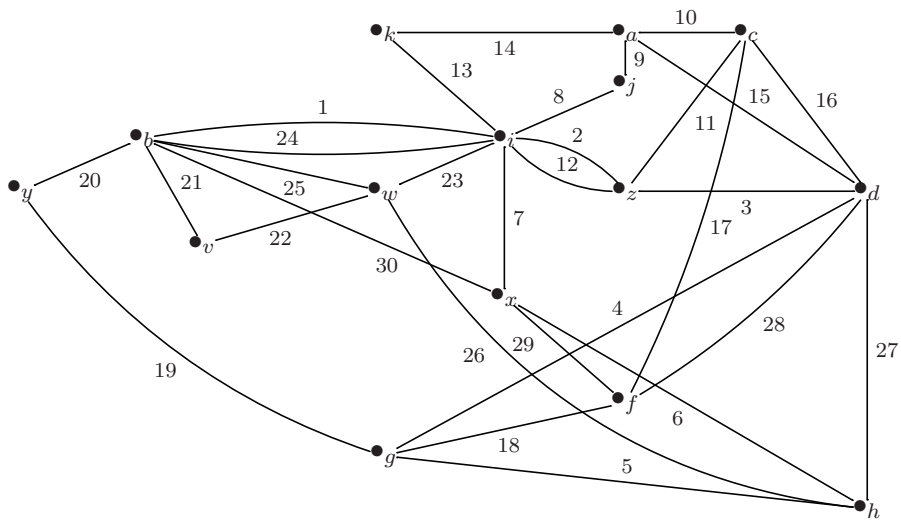
Now we solve the Euler Circuit problem on the expanded graph. By starting with the short b-z path we will easily remove it at the end to solve the Euler Tour problem.



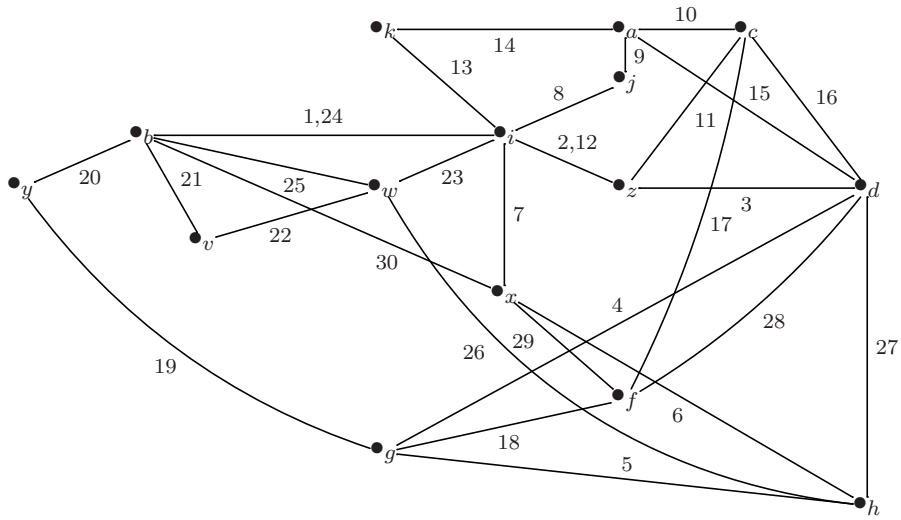
We can keep going from b :



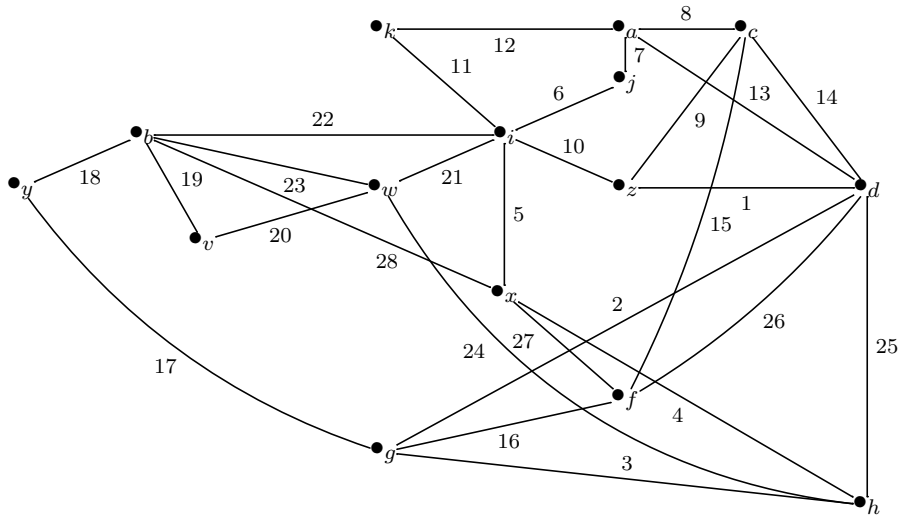
We need to splice in one last cycle (the black):



Here is the solution to the Chinese Postman problem, where just two edges get traversed twice:



For the Euler Tour, just skip the first two edges:



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