0.1. The incidence Function. The definition of a graph needs to have three things: a set of vertices, a set of edges, and something to describe where the edges go. In the book, they give this example,

which they say has vertex set

\[ V = \{a, b, c, d, e\} \]

and edge set

\[ E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\} \]

and where the ends of the edges are given by

\[
\begin{align*}
e_1 &\leftrightarrow (a, b) & e_5 &\leftrightarrow (b, d) \\
e_2 &\leftrightarrow (b, c) & e_6 &\leftrightarrow (d, e) \\
e_3 &\leftrightarrow (c, c) & e_7 &\leftrightarrow (b, e) \\
e_4 &\leftrightarrow (c, d) & e_8 &\leftrightarrow (b, e)
\end{align*}
\]

and where the pairs \((v, w)\) are all unordered pairs. I have two small complaints.

You can’t figure which edge a pair of vertices came from, so the bidirectional arrows are misleading. It is more accurate to say the graph contains this information:

\[
\begin{align*}
e_1 &\leftrightarrow \{a, b\} & e_5 &\leftrightarrow \{b, d\} \\
e_2 &\leftrightarrow \{b, c\} & e_6 &\leftrightarrow \{d, e\} \\
e_3 &\leftrightarrow \{c, c\} & e_7 &\leftrightarrow \{b, e\} \\
e_4 &\leftrightarrow \{c, d\} & e_8 &\leftrightarrow \{b, e\}
\end{align*}
\]

Secondly, the notation \((x, y)\) for an ordered pair is so standard, so it is tricky to use the same notation for an unordered pair. We don’t need any new notation anyway; we are talking about sets that have one or two elements. So, even better is

\[
\begin{align*}
e_1 &\leftrightarrow \{a, b\} & e_5 &\leftrightarrow \{b, d\} \\
e_2 &\leftrightarrow \{b, c\} & e_6 &\leftrightarrow \{d, e\} \\
e_3 &\leftrightarrow \{c, c\} & e_7 &\leftrightarrow \{b, e\} \\
e_4 &\leftrightarrow \{c, d\} & e_8 &\leftrightarrow \{b, e\}
\end{align*}
\]
or

\[
\begin{align*}
e_1 &\mapsto \{a, b\} & e_5 &\mapsto \{b, d\} \\
e_2 &\mapsto \{b, c\} & e_6 &\mapsto \{d, e\} \\
e_3 &\mapsto \{c\} & e_7 &\mapsto \{b, e\} \\
e_4 &\mapsto \{c, d\} & e_8 &\mapsto \{b, e\}
\end{align*}
\]

This table is describing a function, \( \iota \) from \( E \) to \( V \), called the incidence function.

0.2. **Equality.** Two graphs are be **equal** if the have:

(a) the same vertices  
(b) the same edges  
(c) the same incidence information/function

**Example 0.1.** Draw all the possible graphs that have vertex set

\( \{v, w\} \)

and edge set

\( \{e, f\} \).

**Solution:** Each edge needs to be given one or two vertices to which to be incident. The choices are

- just \( v \),
- just \( w \),
- \( v \) and \( w \).

So there are a total of 9 possibilities: