TELLING TREES APART

We’ve seen two of the common methods for showing that two graphs are not isomorphic. One is to look at the degree sequences. The other is to count how many small cycles each graph contains (generally 3-cycles, 4-cycles or 5-cycles). In trees, there are never cycles, so that method is not useful in the context of trees.

If two trees have the same degree sequence, they will have the same number of leaves. What we can check is the “distance” between leaves.

Definition 1. In a tree $T$, if $v$ and $w$ are vertices, then there is a unique path from $v$ to $w$. We call the length of that path the distance from $v$ to $w$. We often denote this $d(v, w)$ or $d_T(v, w)$ if we need to differentiate which tree we are talking about.

For example, given this tree

we have

$$d(b, d) = 1$$

and

$$d(a, f) = 4.$$ 

Here is a second tree with the same degree sequence:

We can differentiate these two trees by looking at the leaves and the distances between all pairs:
We see that one graph has a pair of leaves that are distance 3 apart, while the other does not.

The exact relation between the tables of leaf-to-leaf distances in isomorphic trees is something for you to ponder.