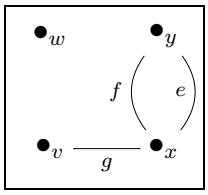
PSEUDOGRAPHS

So what is a multigraph? The short answer is that it is an object just like a graph except that it can have multiple edges between vertices, and the edge have labels so we can identify them. Loops are still not allowed.

Here is picture of a typical small multigraph:

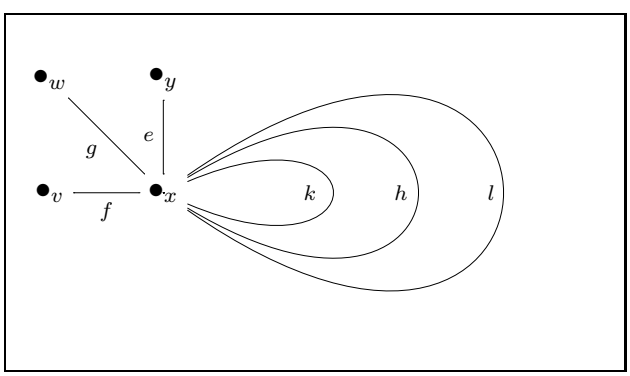
$G =$ 

We can describe G with complete accuracy if we say:

(a) the set of vertices for G is $\{x, y, v, w\}$;
(b) the set of edges for G is $\{e, f, g\}$; and
(c) the edges attach the the edges as follows:
  (a) the edge e is incident to x and y;
  (b) the edge f is incident to x and y;
  (c) the edge g is incident to x and v.

We also are interested in loop-multigraphs or *pseudographs*. (Multi-loop-graphs?) N.B. In many books, the term graph refers to what we will call a loop-multigraph or pseudograph. I won't comment on why the terminology is such a mess.

Here then is a picture of a loop-multigraph:



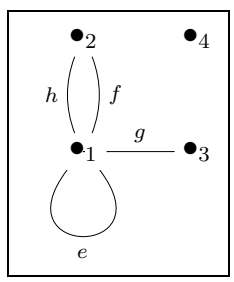
There are many mathematical structures we could use to capture this information. The construction we will use does not start with directed pseudographs.

Definition 1. A pseudograph is an ordered pair $G = (V, P)$ where V is a finite set and P is a set of pairs of the form $(e, \{v, w\})$ where v and w are elements of V and no two of the pairs in P have the same first coordinate. We call e an edge of G and say that e is incident to v and to w. If $v = w$, then the pair is really just $(e, \{v\})$ and we say e as a loop at v.

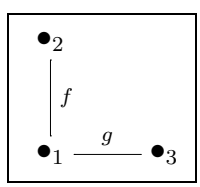
For example,

$$G = \left(\left\{1, 2, 3, 4\right\}, \left\{(e, \{1\}), (f, \{1, 2\}), (g, \{1, 3\}), (h, \{2, 1\})\right\}\right)$$

is the pseudograph we would draw thus:

$G =$ 

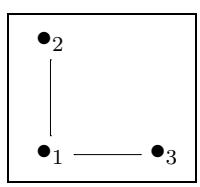
We would like to say that a graph is a pseudograph that has no loops and no multiple edges (meaning there are two or more edges incident to the same $\{v, w\}$). This is not excactly true. For example



is a drawing of the pseudograph

$$\left(\left\{1, 2, 3\right\}, \left\{(f, \{1, 2\}), (g, \{1, 3\})\right\}\right)$$

and



is a drawing of the graph

$$\left(\left\{1, 2, 3\right\}, \left\{(1, 2), (2, 1), (1, 3), (3, 1)\right\}\right).$$

If you think this is a silly distinction, you are correct as far as mathematics goes. We will not worry about which set-theoretical picture is backing up the pictures.

If you are into computers, you may think that the two set-theoretical definitions lead to very different ways to store graphs. However, neither one leads to a very precise data structure, because the primary storage paradigm on a computer is a list (array) and not a set.