PSEUDOGRAPHS

So what is a multigraph? The short answer is that it is an object just like a graph except that it can have multiple edges between vertices, and the edge have labels so we can identify them. Loops are still not allowed.

Here is picture of a typical small multigraph:

\[ G = \begin{array}{c}
\bullet w \\
\bullet y \\
\bullet x \\
\bullet v \\
\end{array}
\]

We can describe \( G \) with complete accuracy if we say:

(a) the set of vertices for \( G \) is \( \{x, y, v, w\} \);
(b) the set of edges for \( G \) is \( \{e, f, g\} \); and
(c) the edges attach the the edges as follows:
   (a) the edge \( e \) is incident to \( x \) and \( y \);
   (b) the edge \( f \) is incident to \( x \) and \( y \);
   (c) the edge \( g \) is incident to \( x \) and \( v \).

We also are interested in loop-multigraphs or pseudographs. (Multi-loop-graphs?) N.B. In many books, the term graph refers to what we will call a loop-multigraph or pseudograph. I won’t comment on why the terminology is such a mess.

Here then is a picture of a loop-multigraph:
There are many mathematical structures we could use to capture this information. The construction we will use does not start with directed pseudographs.

**Definition 1.** A pseudograph is an ordered pair $G = (V, P)$ where $V$ is a finite set and $P$ is a set of pairs of the form $(e, \{v, w\})$ where $v$ and $w$ are elements of $V$ and no two of the pairs in $P$ have the same first coordinate. We call $e$ an edge of $G$ and say that $e$ is incident to $v$ and to $w$. If $v = w$, then the pair is really just $(e, \{v\})$ and we say $e$ as a loop at $v$.

For example,

$$G = \left( \left\{ 1, 2, 3, 4 \right\}, \left\{ (e, \{1\}), (f, \{1, 2\}), (g, \{1, 3\}), (h, \{2, 1\}) \right\} \right)$$

is the pseudograph we would draw thus:

![Pseudograph](image)

We would like to say that a graph is a pseudograph that has no loops and no multiple edges (meaning there are two or more edges incident to the same $\{v, w\}$). This is not exactly true. For example

![Graph](image)

is a drawing of the pseudograph

$$\left( \left\{ 1, 2, 3 \right\}, \left\{ (f, \{1, 2\}), (g, \{1, 3\}) \right\} \right)$$

and

![Graph](image)

is a drawing of the graph

$$\left( \left\{ 1, 2, 3 \right\}, \left\{ (1, 2), (2, 1), (1, 3), (3, 1) \right\} \right).$$
If you think this is a silly distinction, you are correct as far as mathematics goes. We will not worry about which set-theoretical picture is backing up the pictures.

If you are into computers, you may think that the two set-theoretical definitions lead to very different ways to store graphs. However, neither one leads to a very precise data structure, because the primary storage paradigm on a computer is a list (array) and not a set.