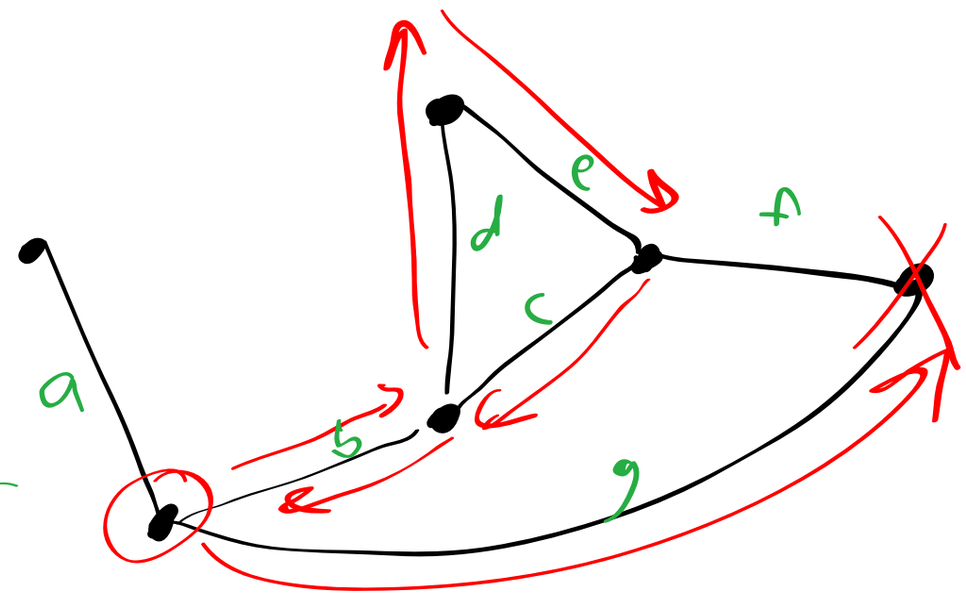
Wed. 1/30, Lecture #7

Walks, trails, paths ... (Read more defns in book)



b, d, e, c, b, g.

<u>Def'n</u> Given a Graph G, a **walk** is a succession of edges, each incident to a vertex incident to the previous edge:

$$(v_1v_2, v_2v_3, \ldots, v_{n-2}v_{n-1}, v_{n-1}, v_n)$$

Caution: We'll need a fix on this definition for Graphs with multiple edges.

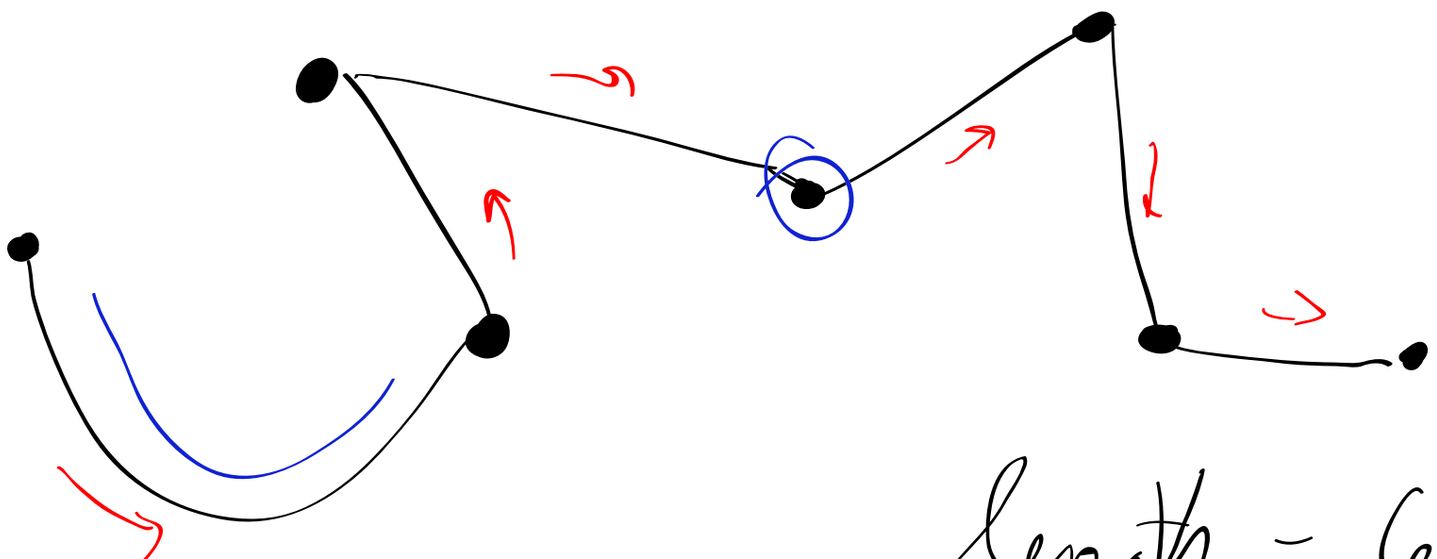
Trail. A trail is a walk with distinct edges.

Path. A path is a walk with distinct vertices.

Q: Is every path a trail?

edges: $v_i v_j$, ..., $v_i v_j$

If a path is $v_1 v_2, \ldots, v_{n-1} v_n$, the length is $\cancel{n}$. $n-1$



length = 6.

vw, wq

length is # of edges

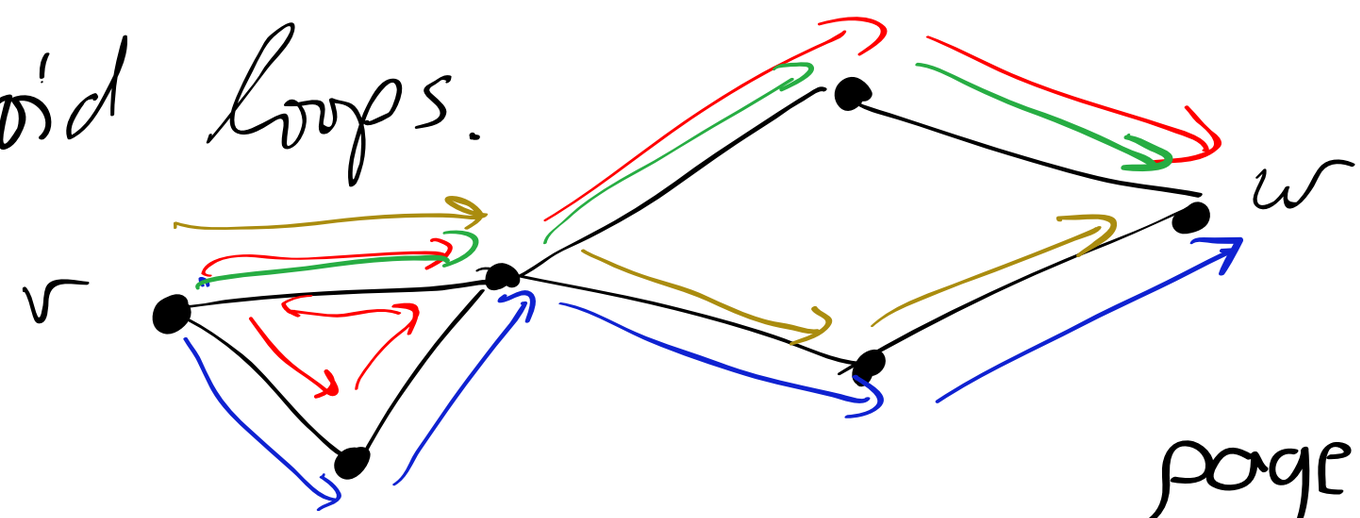
Q: If there is a walk "from v to w", is there a path "from v to w"?

A: Yes.

Q: How to prove this?

Idea. Find "the shortest" walk. Is it not a path.

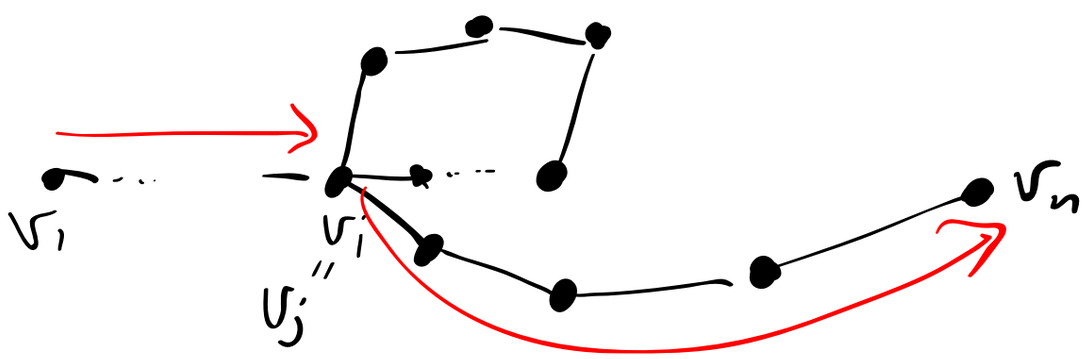
Idea: Avoid loops.



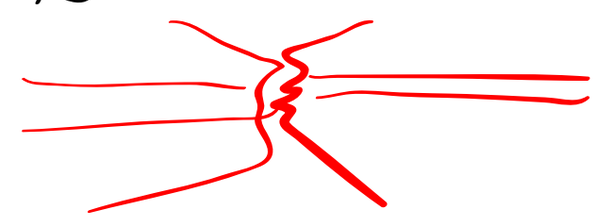
pf: Suppose there is a walk in G from v to w.
Consider a walk of shortest length W:

$$v_1v_2,\ v_2v_3,\ \ldots,\ v_{n-1}v_n \quad (\text{length } n-1)$$

Suppose this were not a path.
This would mean $v_i = v_j$ for some $i < j$



So there is a walk from $v = v_1$ to $v_n = w$ that is shorter than $n-1$.



Conclude that W is a path. $\square$

Bipartite Graphs

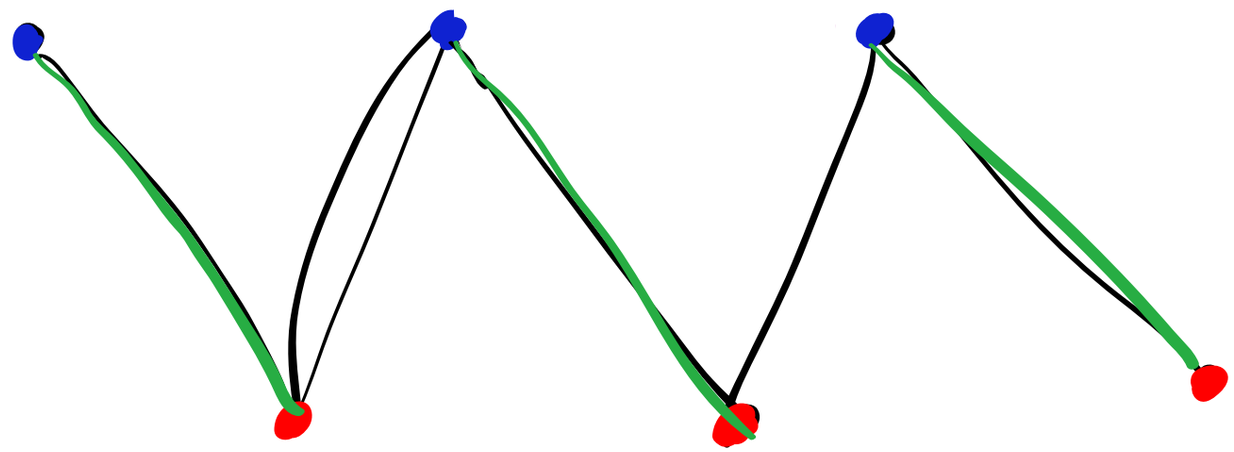
Def. A Graph G is bipartite if, there are subsets B and R of $V(G)$ with:

$$B \cap R = \emptyset;$$

$$B \cup R = V(G);$$

and every edge in G is incident to a vertex in B and a vertex in R;

and $B \neq \emptyset$ and $R \neq \emptyset$.



A bipartite graph is a graph whose vertices <u>can be</u> so partitioned.

The partition <u>is not</u> additional data comprising part of a bipartite graph.