Wed. 1/30, Lecture #7

Walks, trails, paths ... (Read more definitions in book)

**Defn** Given a Graph $G$, a walk is a succession of edges, each incident to a vertex incident to the previous edge:

$$ (V_1V_2, V_2V_3, \ldots, V_{n-2}V_{n-1}, V_{n-1}V_n) $$

**Caution:** We'll need a fix on this definition for graphs with multiple edges.
Trail. A trail is a walk with distinct edges.

Path. A path is a walk with distinct vertices.

Q: Is every path a trail?

edges: $v_i v_j$, $\ldots$, $v_i v_j$

If a path is $v_1 v_2, \ldots, v_{n-1} v_n$, the length is $n - 1$
length = 6.

vw, wq

length is # of edges
Q: If there is a walk "from v to w", is there a path "from v to w"?

A: Yes.

Q: How to prove this?

Idea: Find the shortest walk. Is it not a path?

Idea: Avoid loops.
pf: Suppose there is a walk in $G$ from $v$ to $w$. Consider a walk of shortest length:

$v, v_2, v_3, \ldots, v_{n-1}, v_n$ (length $n-1$)

Suppose this were not a path.

This would mean $v_i = v_j$ for some $i < j$.

So there is a walk from $v = v_i$ to $v_n = w$ that is shorter than $n-1$.

Conclude that $W$ is a path. □
Bipartite Graphs

Def. A graph $G$ is bipartite if, there are subsets $B$ and $R$ of $V(G)$ with:

$$B \cap R = \emptyset;$$

$$B \cup R = V(G),$$

and every edge in $G$ is incident to a vertex in $B$ and a vertex in $R;$

and $B \neq \emptyset$ and $R \neq \emptyset.$
A bipartite graph is a graph whose vertices can be so partitioned that partition is not additional data comprising part of a bipartite graph.