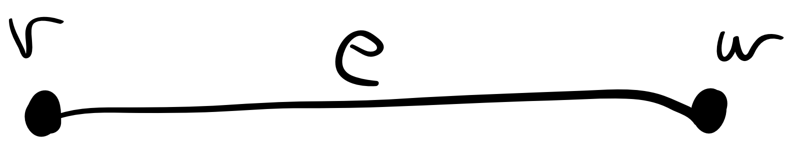
Lecture 6. Monday. 1/28.

Incident & Adjacent.



v is adjacent to w

w is adjacent to v

e is incident to v

e is incident to w

v is incident to e

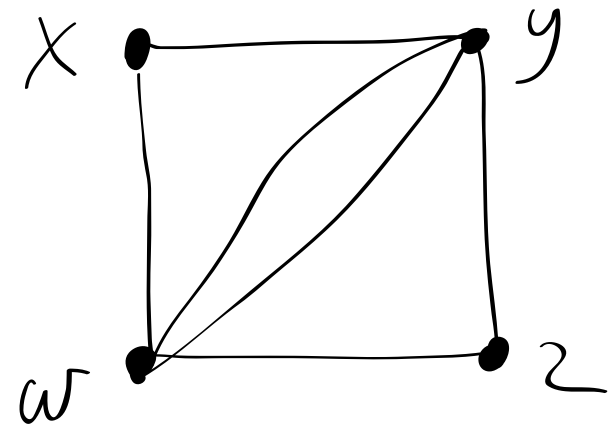
w is incident to e

Adjacency Matrix & Incidence Matrix

Nice one.

An Adjacency Matrix: Example First.

G:



1st order the vertices x, y, z, w

An adjacency matrix of a graph is always symmetric

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$$

x, y: how many edges are between them

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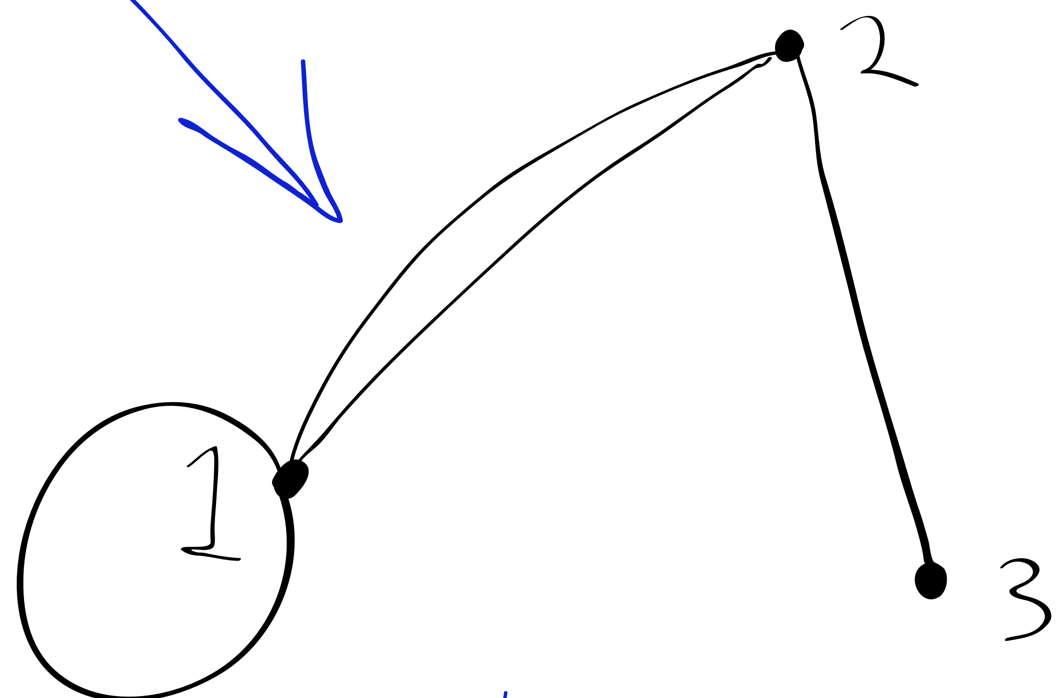
$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

What is the graph?



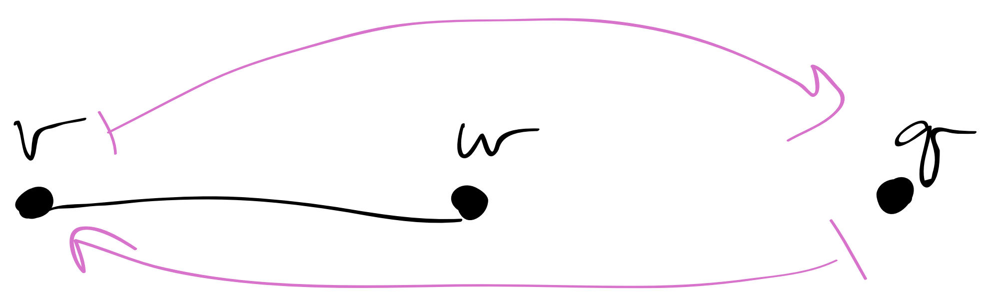
$$\begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Another Adj. Matrix.

order 2, 1, 3

Isomorphism / counting problem

$n = 3, \quad m = 1.$



||

order v, w, q

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

order v, w, q

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$R1 \leftrightarrow R3$ & $C1 \leftrightarrow C3$

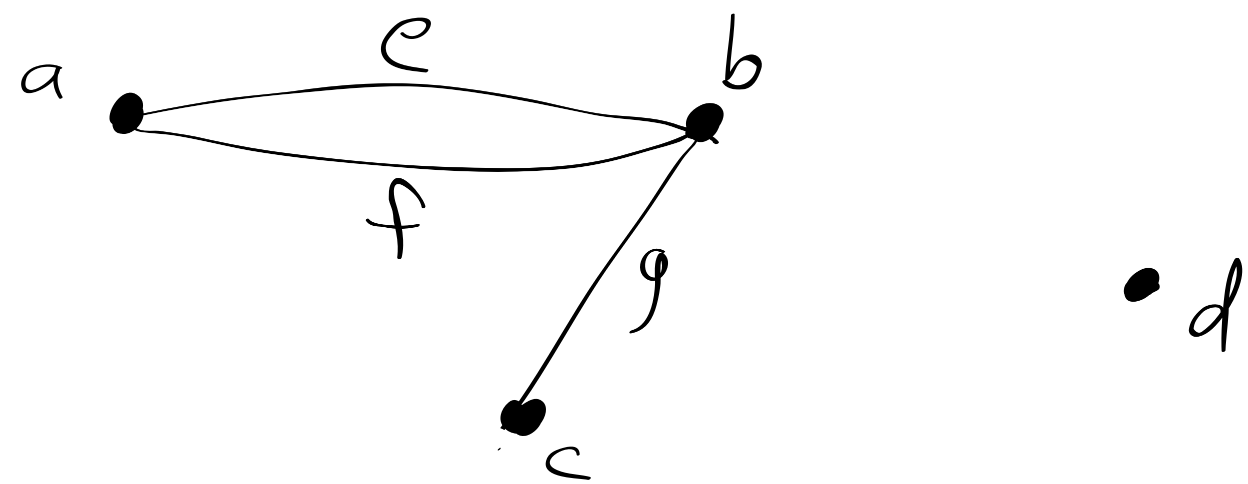
$$\text{LHS} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{RHS}$$

Q: Why is checking (with row/column permutations) Adj. matrices <u>not</u> a fast way to count non-iso. graphs for fixed n, m.?

Time $\approx n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Incidence Matrix :



Need to order both the vertices and the edges :

a, b, c, d || e, f, g

$a \equiv R1$

$$\begin{array}{c|c} & \begin{array}{ccc} e & f & g \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Adj. matrix e,f

$$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$