Lecture 38

About Discrete Math, 327, Fall.

$\nexists$ too much overlap with Math 318

Finite Algebraic Systems will be studied.

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- Degree of a vertex, regular graph, even and odd vertices.
- Degree sequences. The handshake lemma.
- Isomorphism.
- Classifying small graphs.
- Adjacency matricies
- Incidence matrices.
- Walks, Trails, Paths.
- Bipartite graphs.
- Cube graphs. The Petersen graph. Trees. Cycles.
- Rooted trees.
- Directed graphs
- Orientability. Bridges.
- Adjacency and Incidence matrices for digraphs
- Counting digraphs
- In-degree and out-degree sequences.
- Finite State Automata and their languages.
- Euler circuits.
- Edge traceable graphs
- Weighted digraphs.
- Minimal cost paths.
- Longest paths.
- Degrees of connectedness. κ λ.
- Cut-sets and vertex-cut-sets.
- Menger's Theorem
- Planar graphs. Euler's theorem.
- Cololaries to Euler's Thereom
- (have degree <= 5 vertex)
- (m <= 3n - 6) & Tori
- Graphs drawn on Klein bottles.

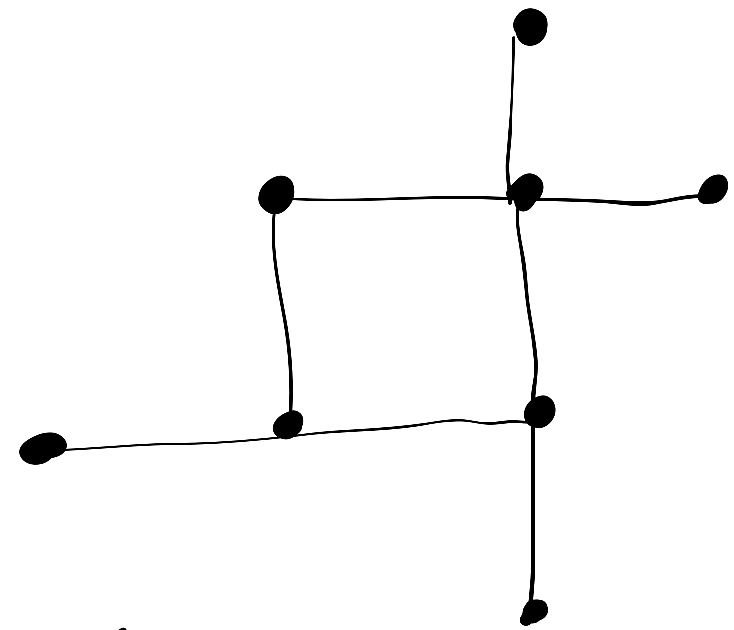
Dual Graphs

Final will evenly cover all parts of the class.

No applications except FSA

Final. The final exam is Monday, May 6, 10:00-12:00. Web student's not attending this exam need to set a time with me.

#1 Consider G:

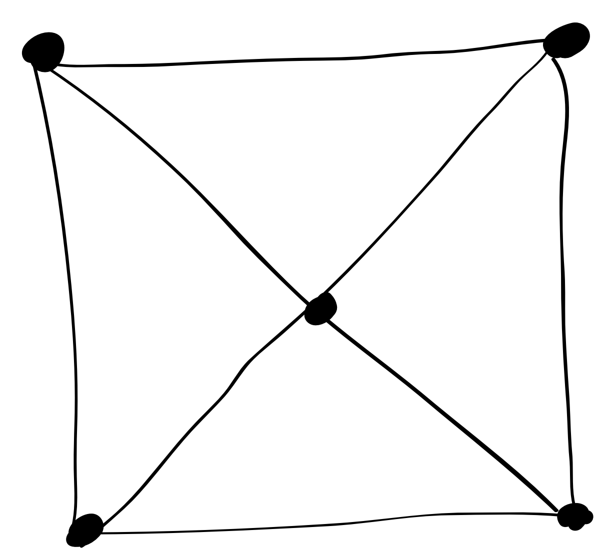


Some sample problems.

(a) What is the chromatic number of G?

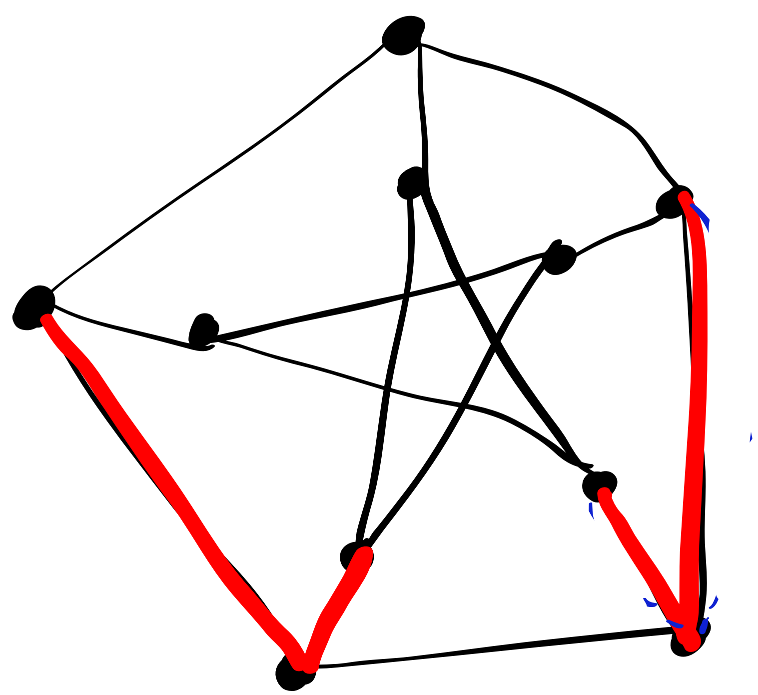
(b) How many ways can G be colored with k-colors, when k is the chromatic number of G?

#2. Let H be

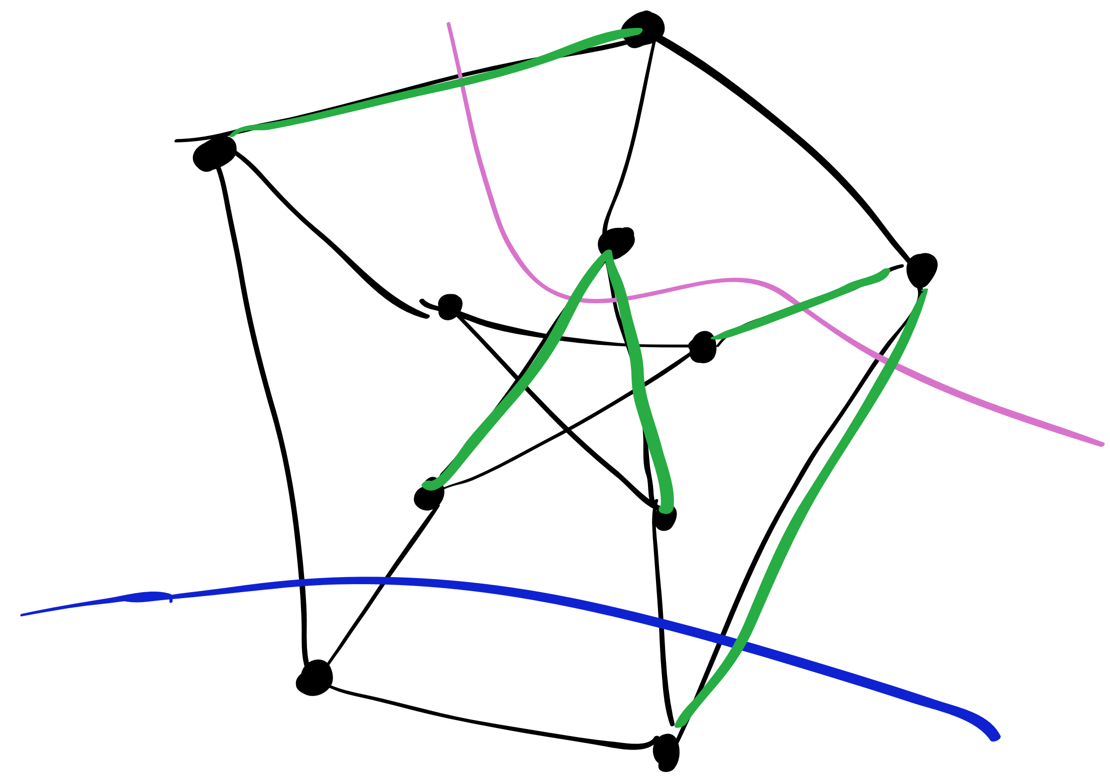


Is the dual of H edge-traceable?

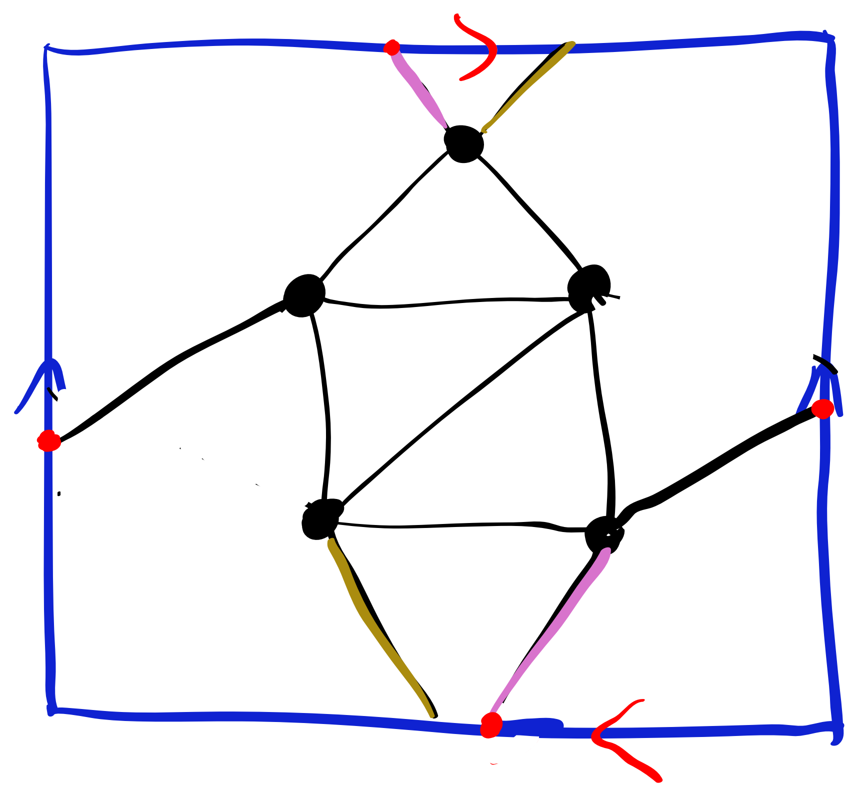
edge-cut-sets

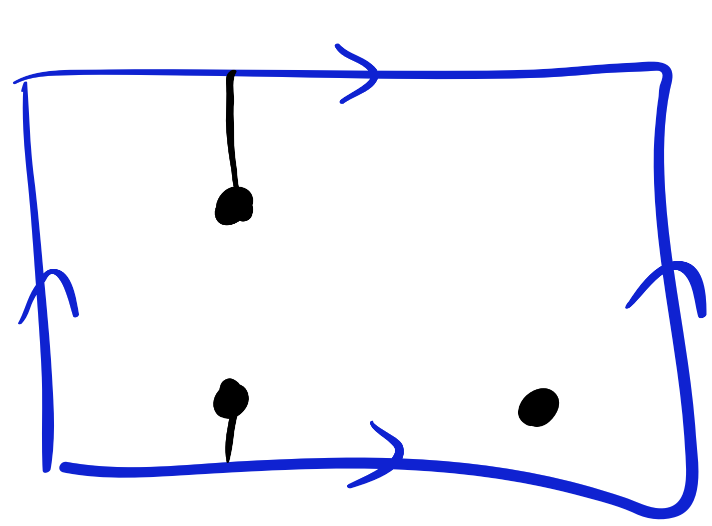


Find an edge-cut-set with ~~4~~ ~~5~~ vertices.

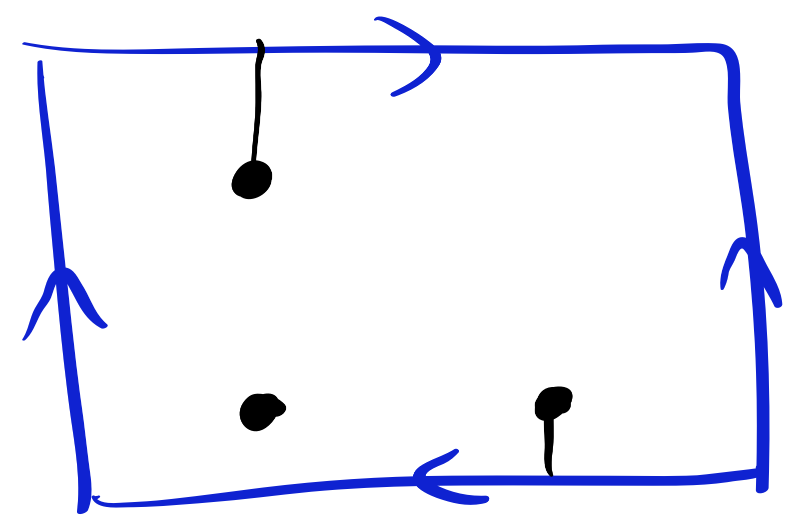


Draw K_5 on a Torus.





Torus



Klein Bottle

Show no simple graph G of 11 vertices can be planar if $\overline{G}$ is planar.

$G.$ $\quad n = 11$, $\; m = ?$

$\overline{G}$ $\quad n = 11$, $\; m' = 55 - m$

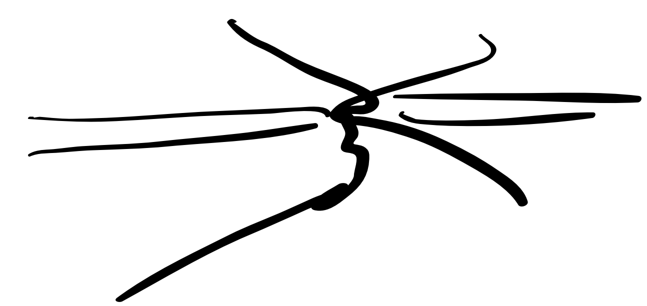
$$\frac{11 \cdot 10}{2} = 55$$

Cor. to Euler:

If both are planar $\rightarrow$ $\begin{cases} m \leq 3 \cdot 11 - 6 = 27 \\ m' \leq 3 \cdot 11 - 6 = 27 \end{cases}$

$+$

$$55 = m + m' \leq 54$$

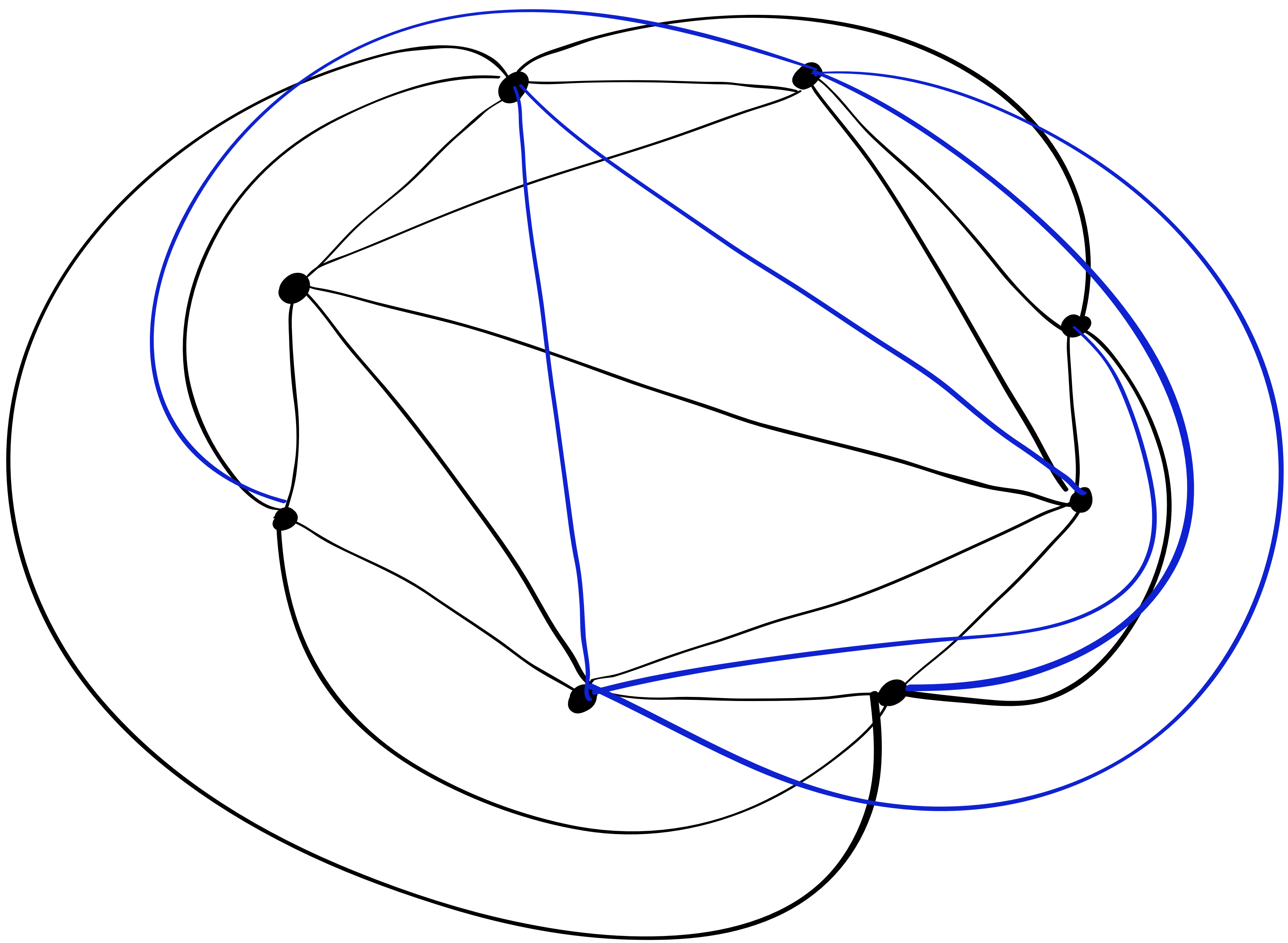


$\underline{n=8}$: $G, \bar{G}$ planar: $m + m' = \frac{8 \cdot 7}{2} = 28$

$m \leq 3 \cdot 8 - 6 = 18$

$m' \leq 18$

G with $\simeq 18$ edges:



~~etc.~~

dang!

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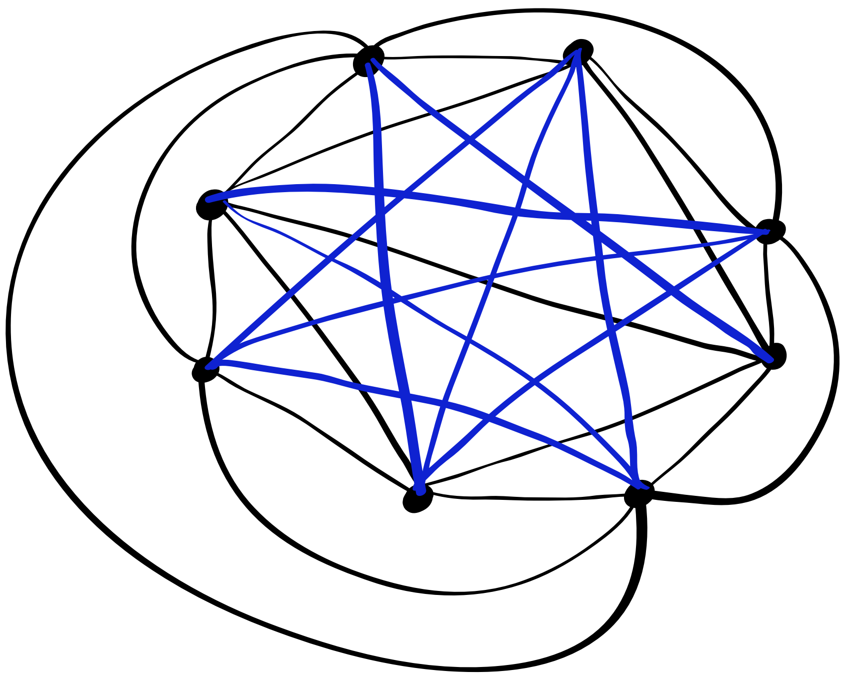
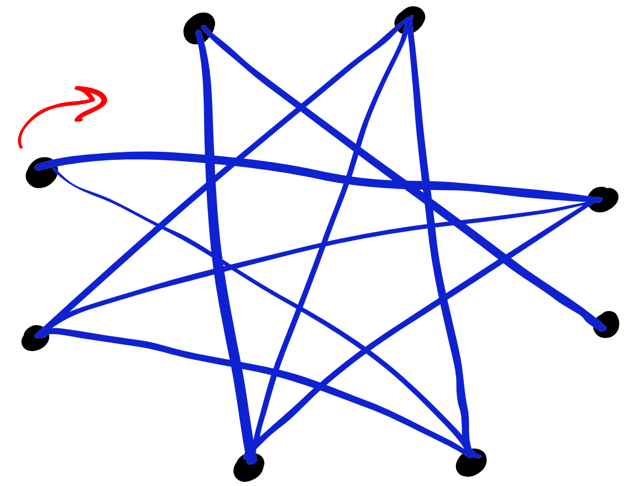
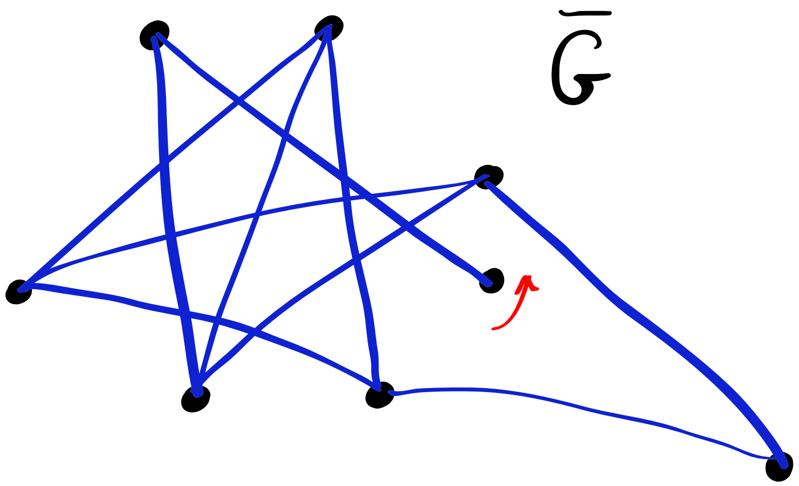


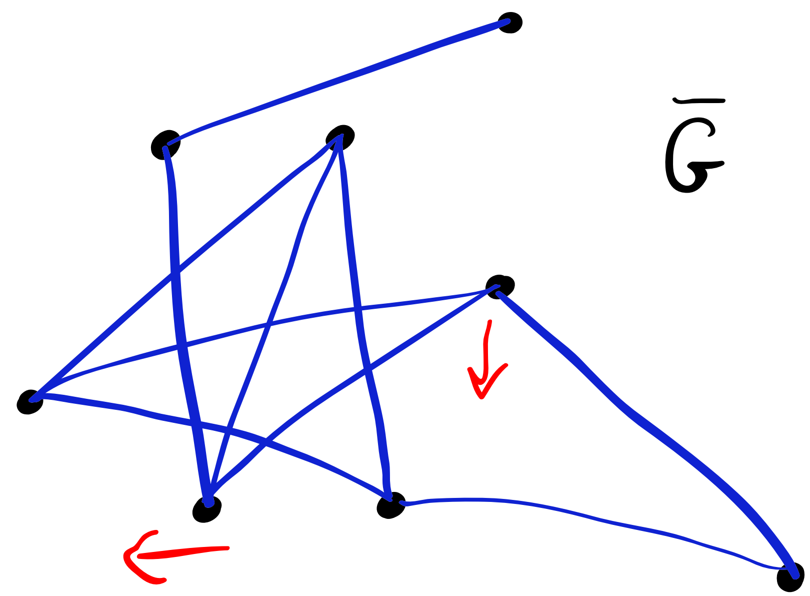
$\bar{G}$



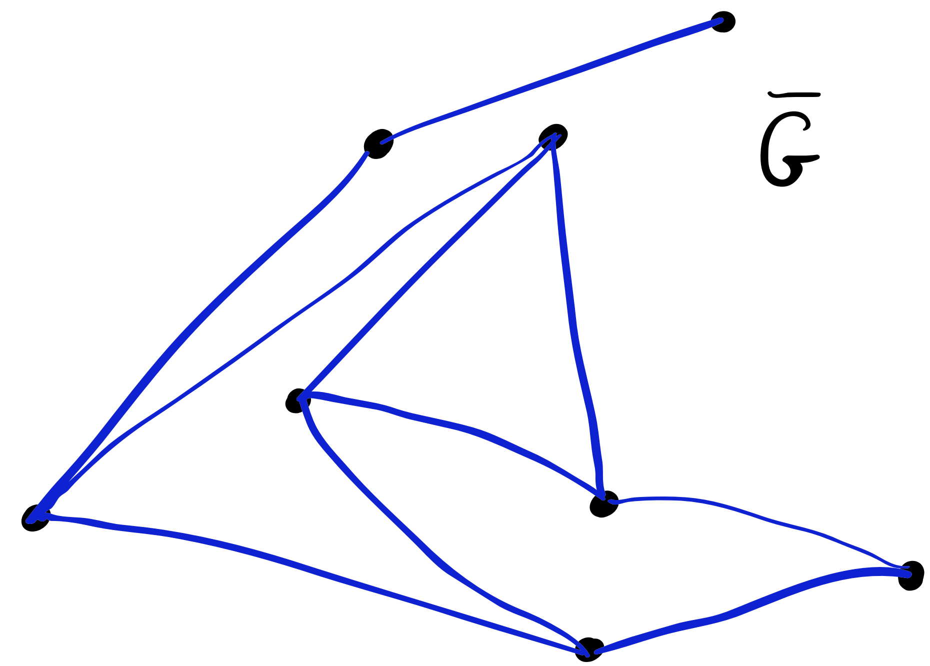
$\bar{G}$



$\bar{G}$

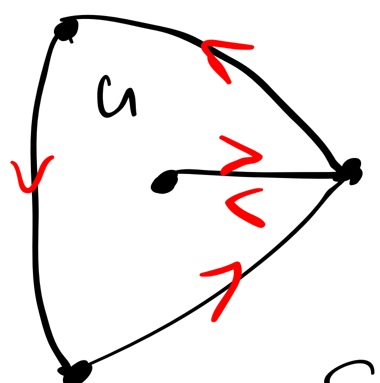


$\bar{G}$



Girth = length of shortest cycle in G. if G has girth g and is planar (simple, connected) then $m \le \frac{(n-2)g}{g-2}$.

The boundary of a face is closed path.



Therefore, if u, a face, has degree d, then $d \ge g$.

So $2m = \sum_{u \text{ a face}} \deg(u) \ge \sum g = fg$.

$\therefore f \le \frac{2m}{g}$. $2 = n - m + f \le n - m + \frac{2m}{g}$.

$\Longrightarrow m - \frac{2m}{g} \le n - 2 \Longrightarrow m \le g\frac{n-2}{g-2}$.