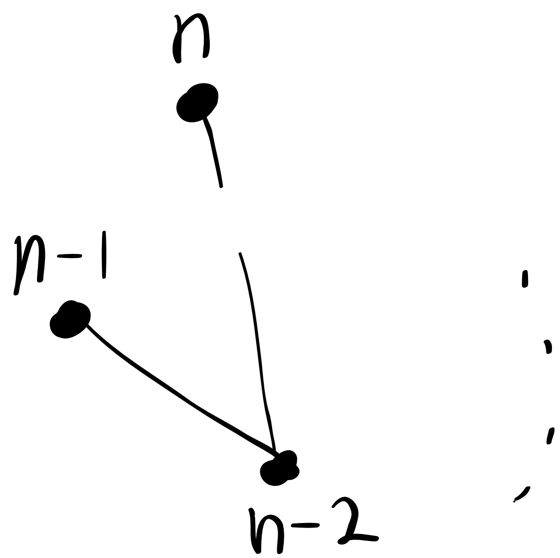
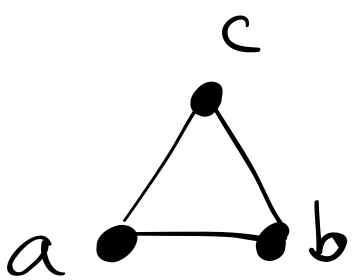
Lecture 36. Details regarding counting colorings of graphs.

$G = K_k$

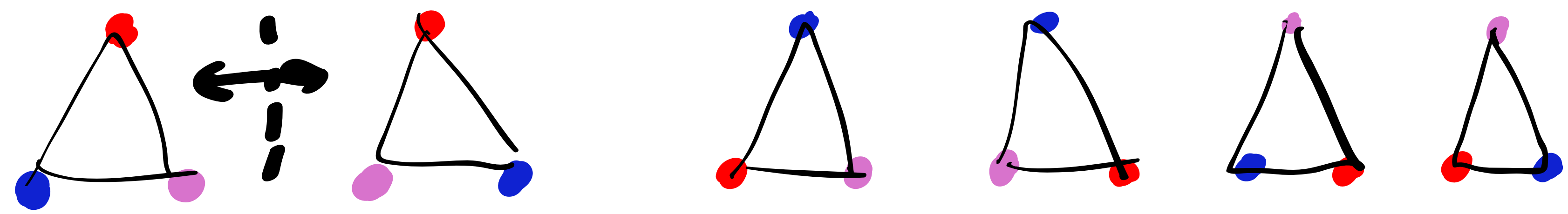


$P_G(n) = n(n-1)\cdots(n-k)$

$P_G(k) = k!$

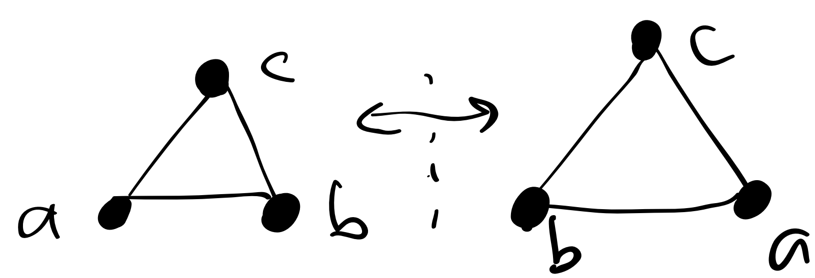
K_3 with 3 colors | 

6 colorings

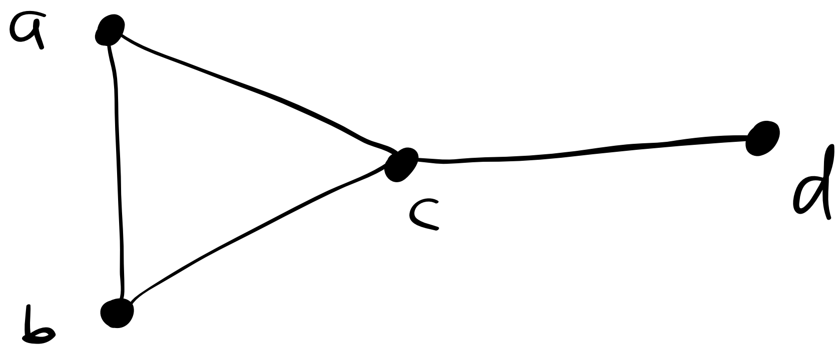


$a \mapsto B$, $b \mapsto P$, $c \mapsto R$; $a \mapsto P$, $b \mapsto B$, $c \mapsto R$; ...

∃ "1 way" to color K_3: give each vertex its own color.



G:



There is a symmetry ($\cong$ of G with G)

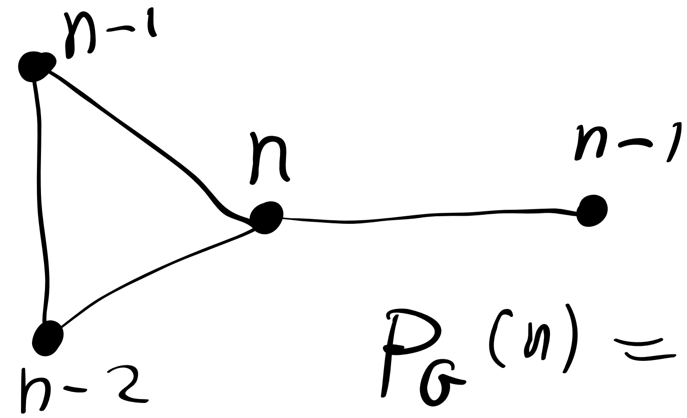
$a \mapsto b$

$b \mapsto a$

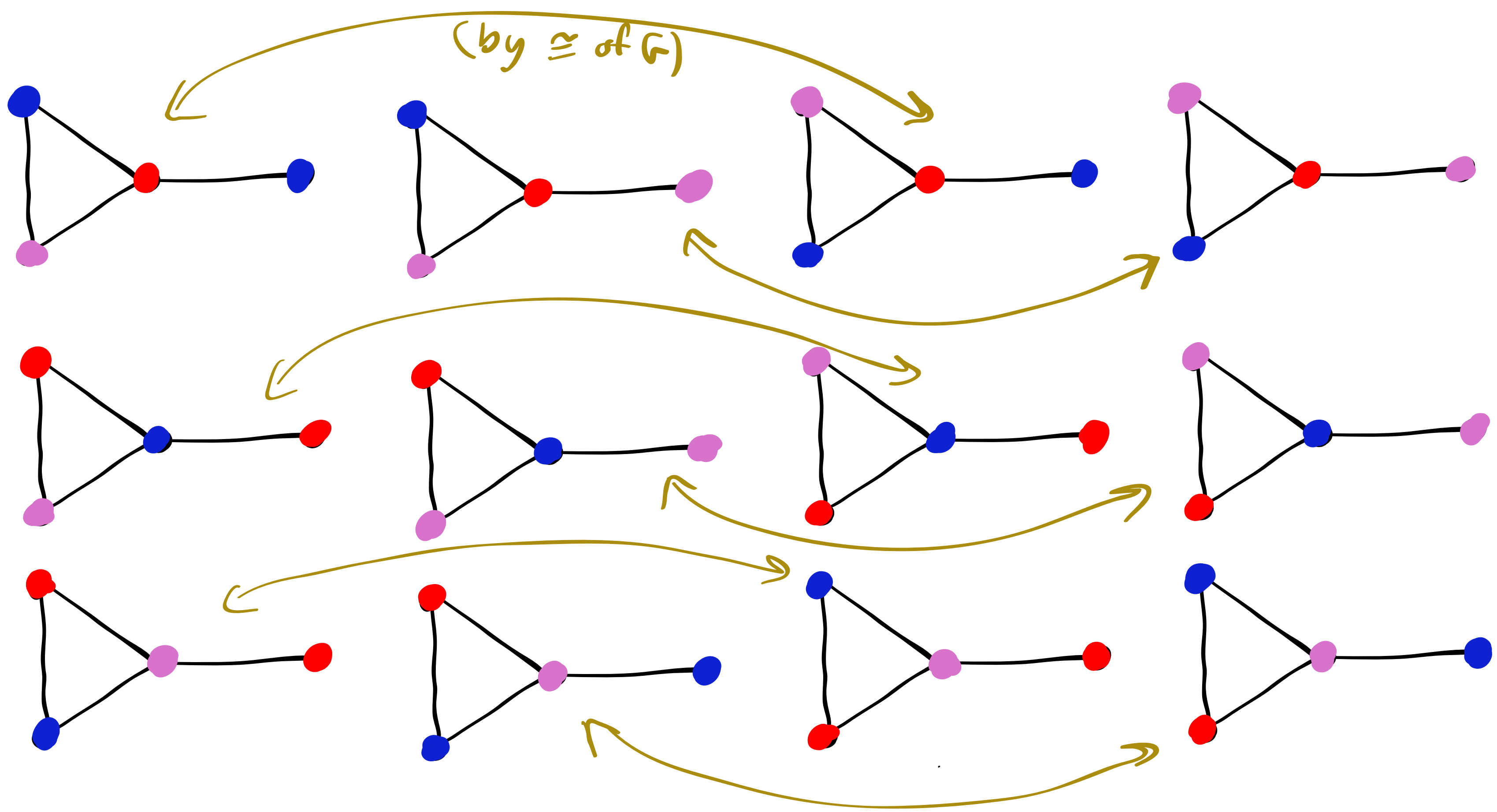
$c \mapsto c$

$d \mapsto d$

$P_G(n)$?

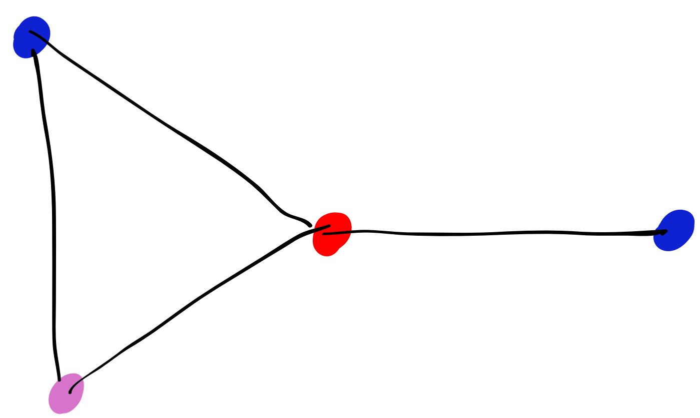


$P_G(n) = n(n-1)^2(n-2)$. $\chi(G) = 3$. $P_G(3) = 12$.

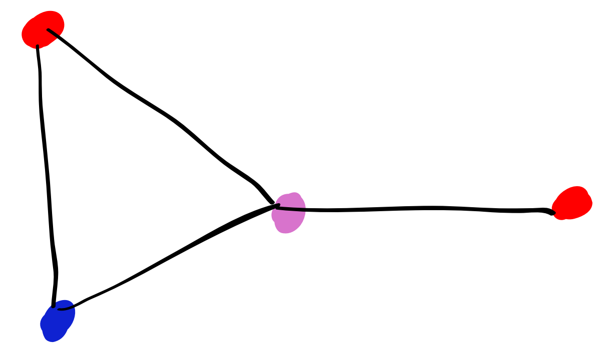


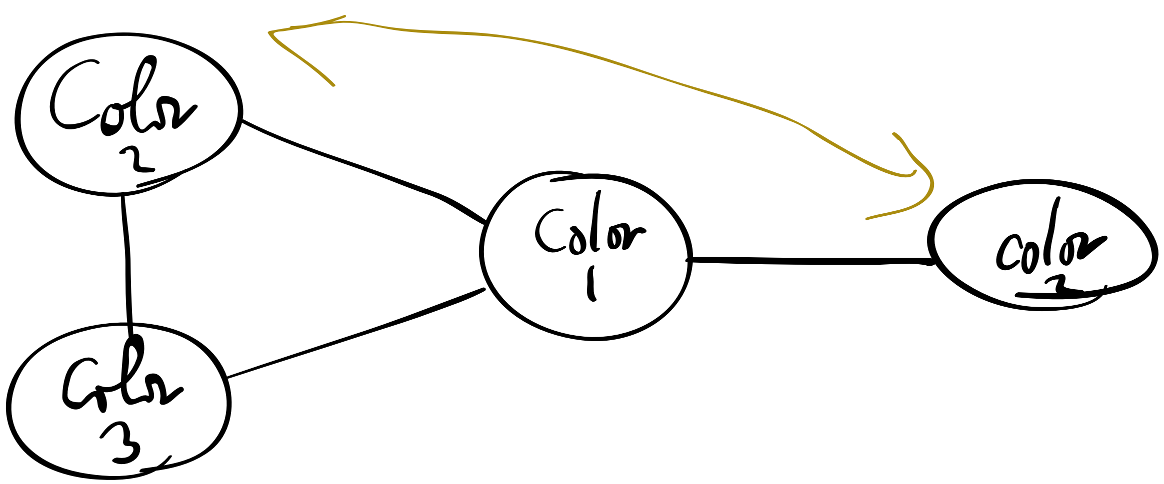
Consider colorings "equivalent" if a symmetry of G takes one coloring to the other.

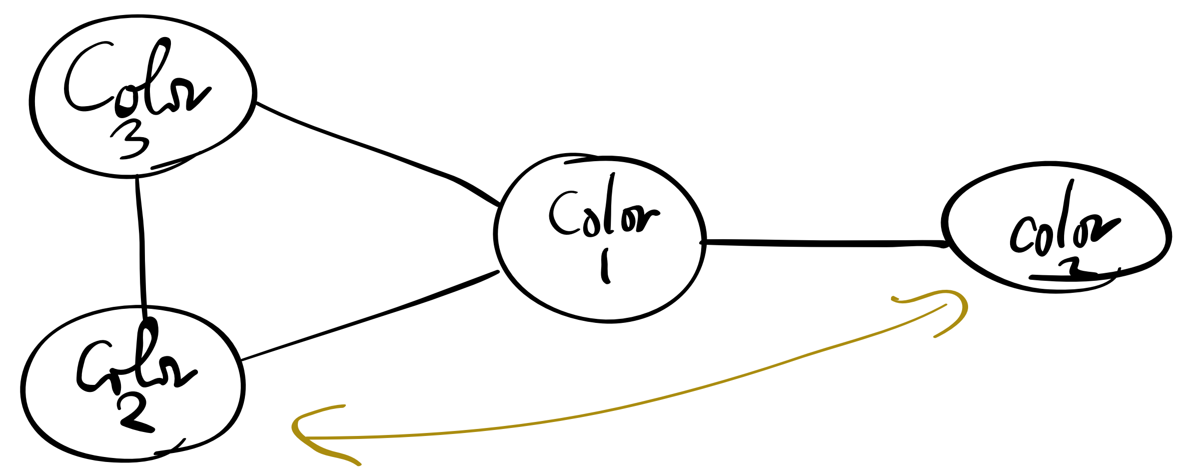
This G has 6 non-equivalent colorings

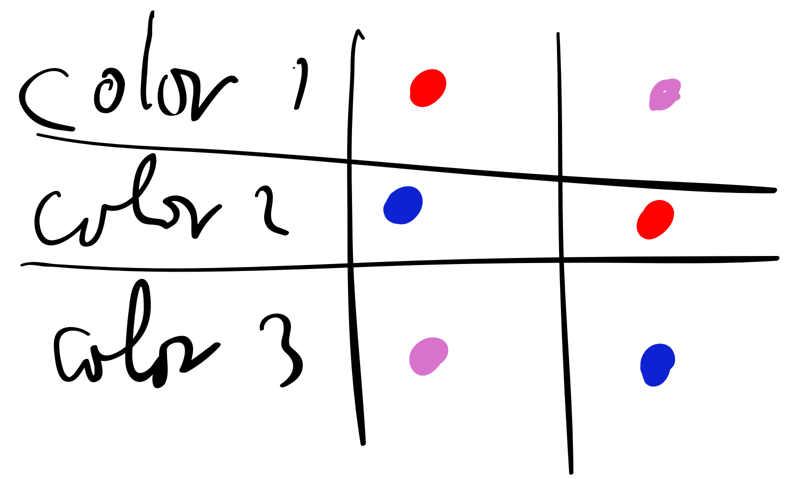


≡ ? ≡



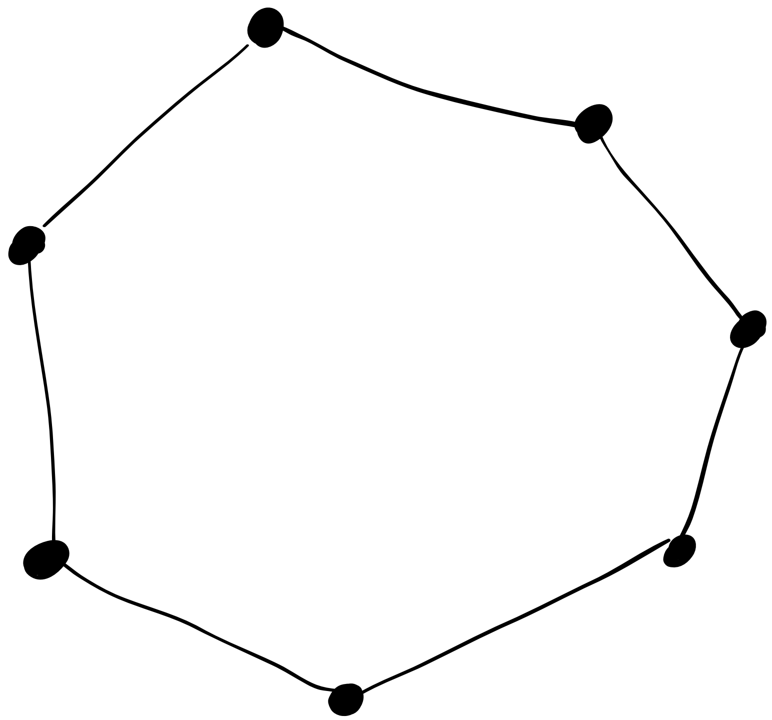




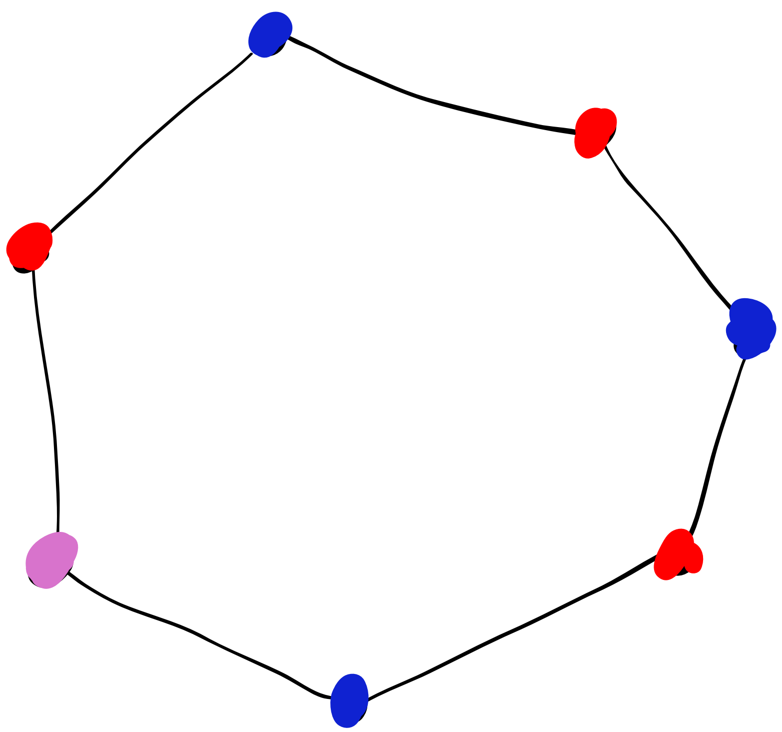


"One way" to color:

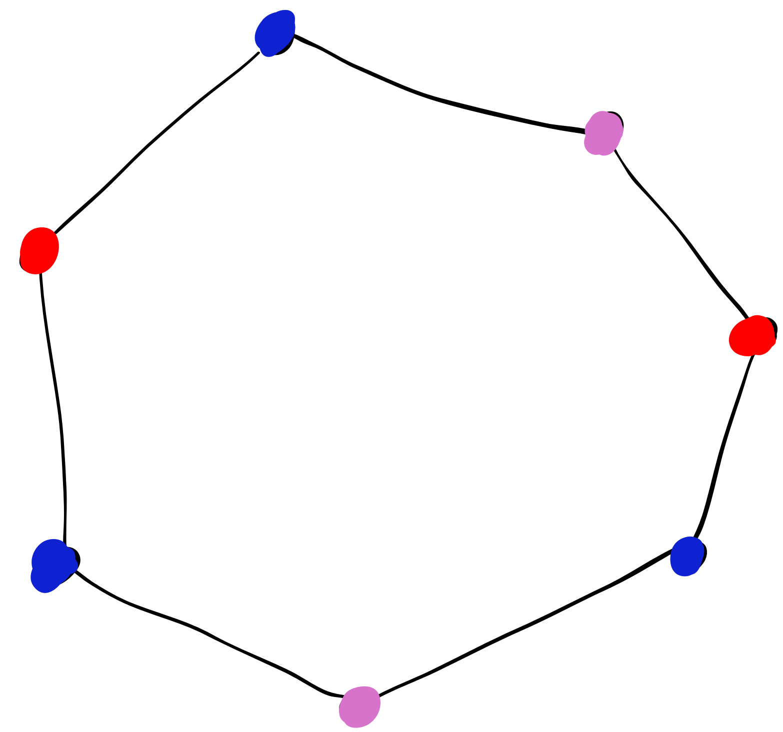
Put a color in middle,
put a color on end,
Repeat on one remaining vertex,
use third color for the last.



This 7-cycle has at least two "truely different" ways to be colored with 3 colors



One color only is used once.



every color appears at least twice.