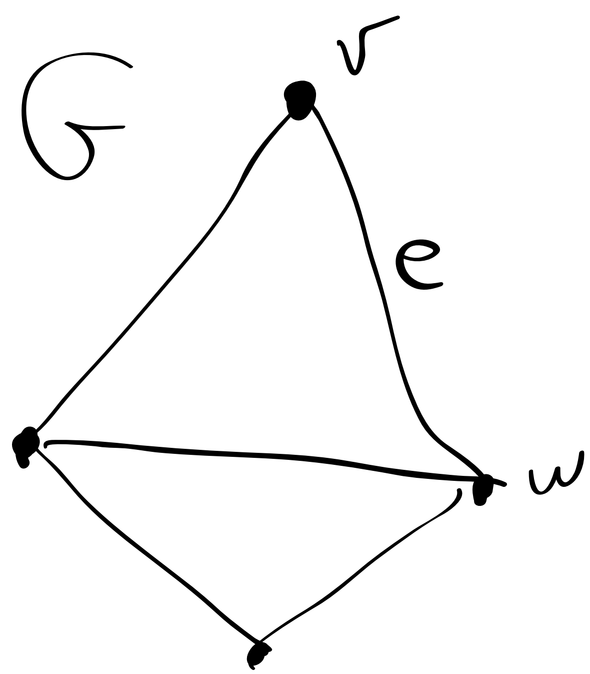
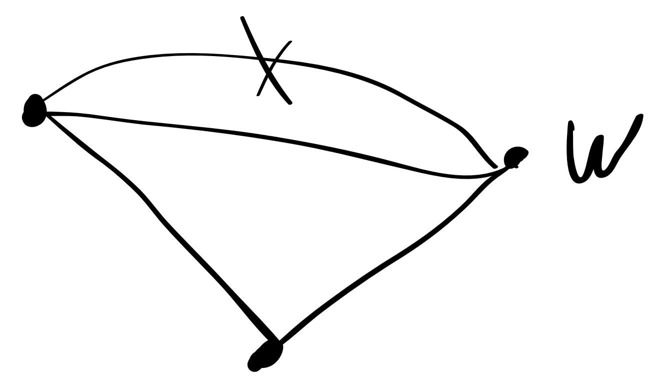
Lecture 35.

G a graph: $G - \{e\}$: same vertices, edge e removed.

$G \cdot e$: contract an edge — then "remove" multiple edges;

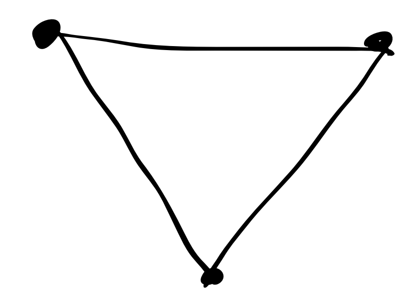
Example: G





e, v, w collapse to form W

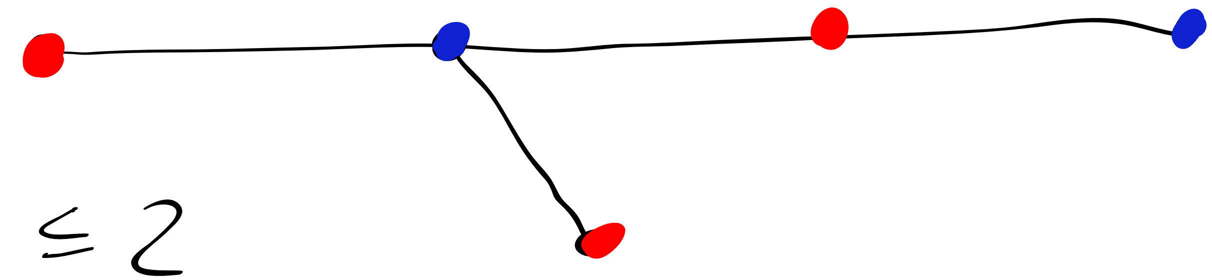
$G \cdot e$



remove one of the multiple edges

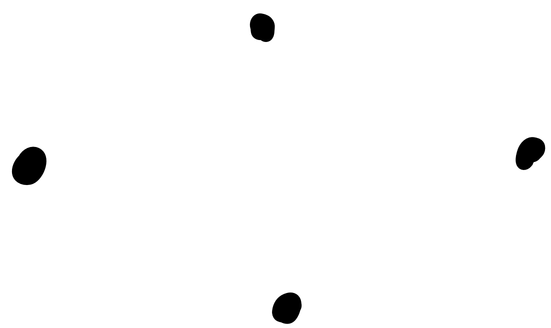
Def: G is **k-colorable** if it is possible to assign to each vertex one out of k-lables s.t. no adjacent vertices have the same lable.

Def: The smallest k for which G is k-colorable is denoted $\chi(G)$, the **chromatic number** of G.

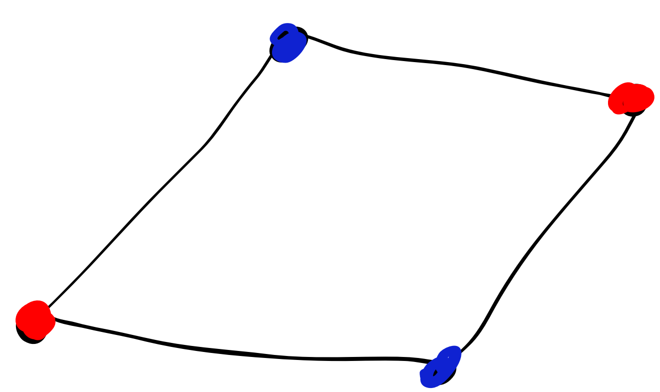


G bipartite $\Longrightarrow$ $\chi(G) \leq 2$.

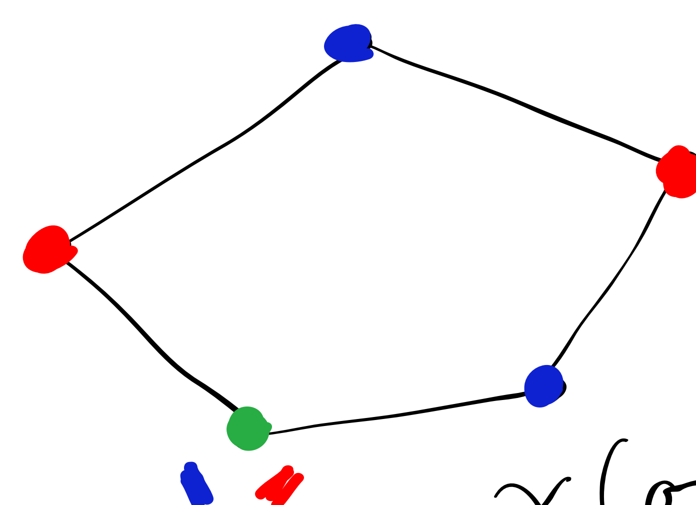
What does $\chi(G) = 1$ mean?

G:  $\chi(G) = 1$.

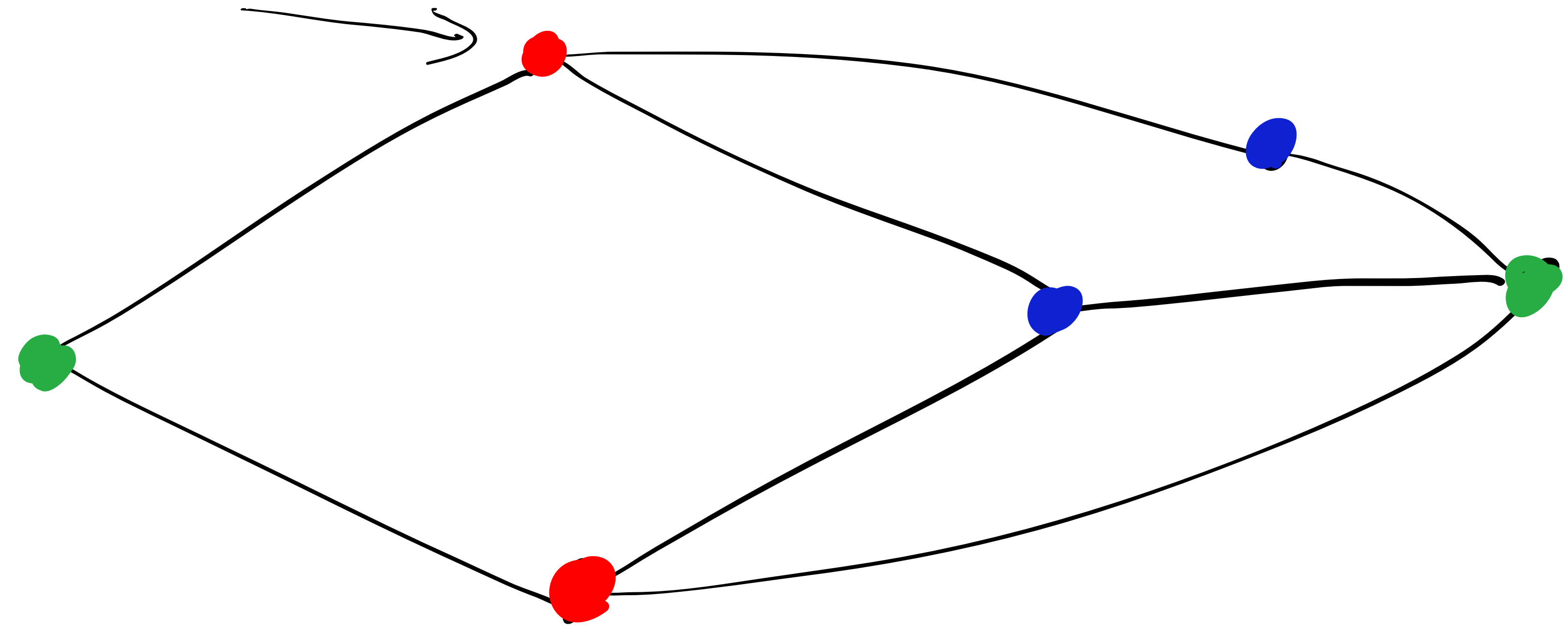
All trees with ≥ 2 vertices have $\chi(G) = 2$.



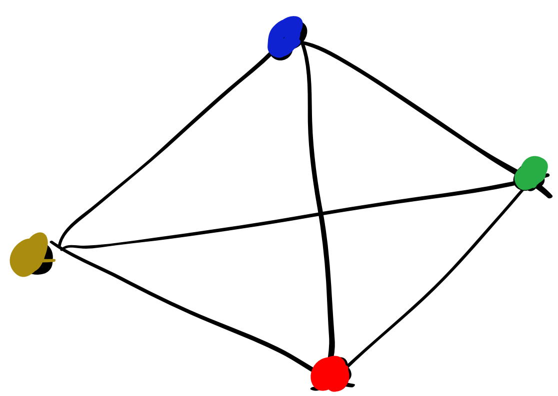
$\chi(\text{even cycle graph}) = 2$



$\chi(\text{odd cycle graph}) = 3$



K_4

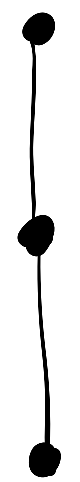


$$\chi(K_n) = n.$$

To make induction work, sometimes study a more complicated topic.

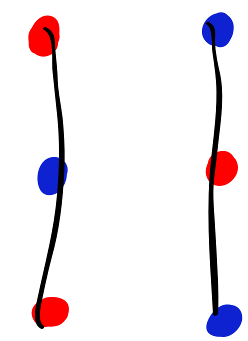
Let's actually count the colorings

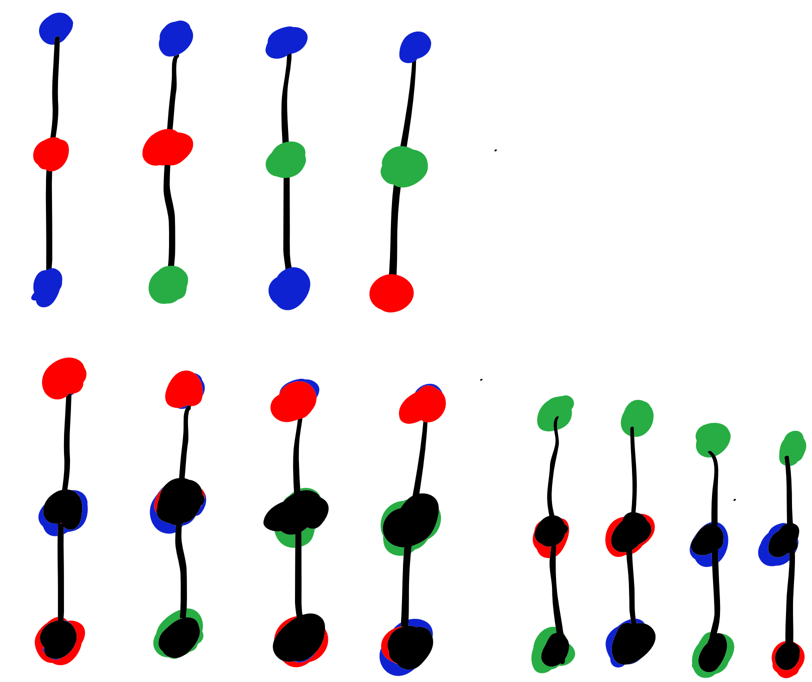
G



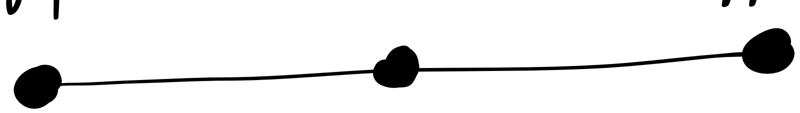
$\chi(G) = 2$

0 colors	1 color	2 colors	3 colors	n
0	0	2	~~4~~ 12	$n(n-1)^2$

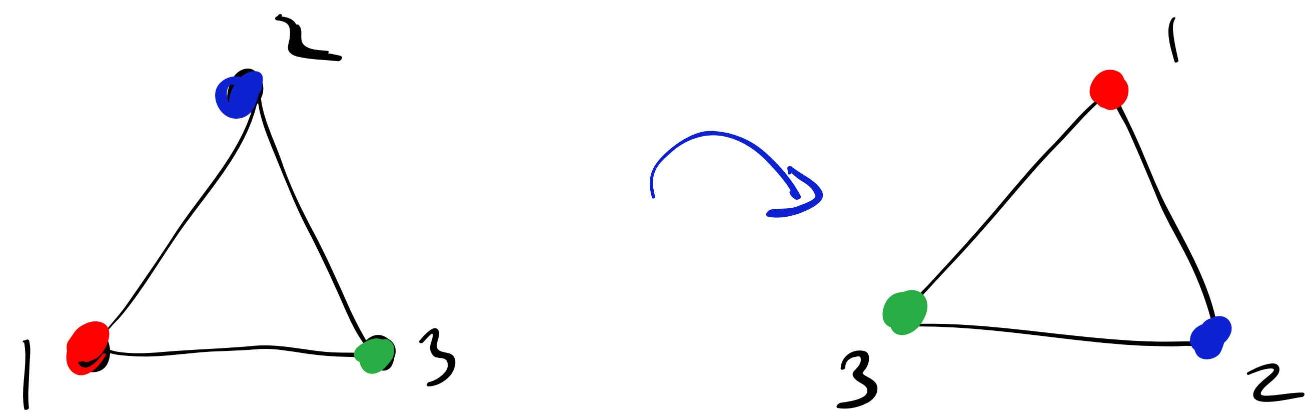




$n \qquad n-1 \qquad n-1$ ← choices of color on that vertex.



pick first — then this — this last

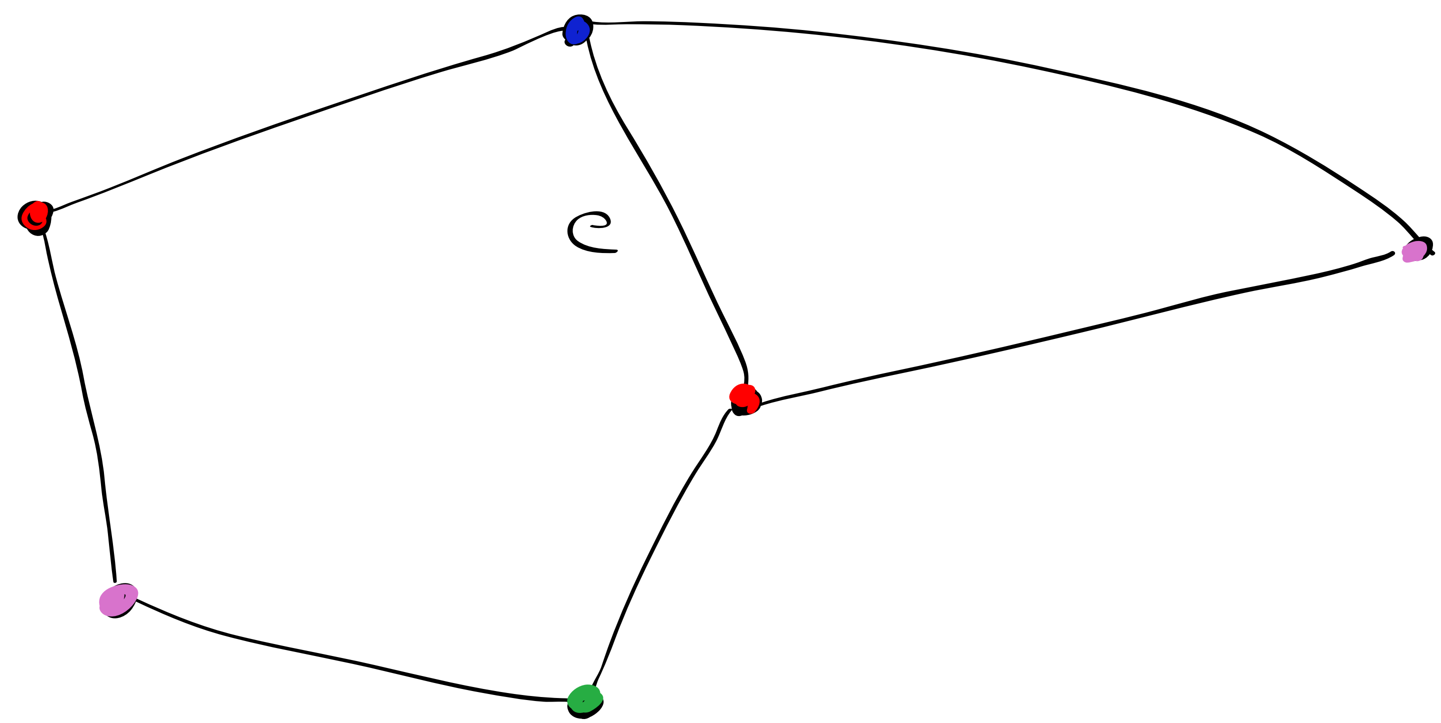


Same coloring, using different drawing of G.

Define: Given G, for $n = 1, 2, \ldots$, let $P_G(n)$ equal the number of colorings of G using n colors.

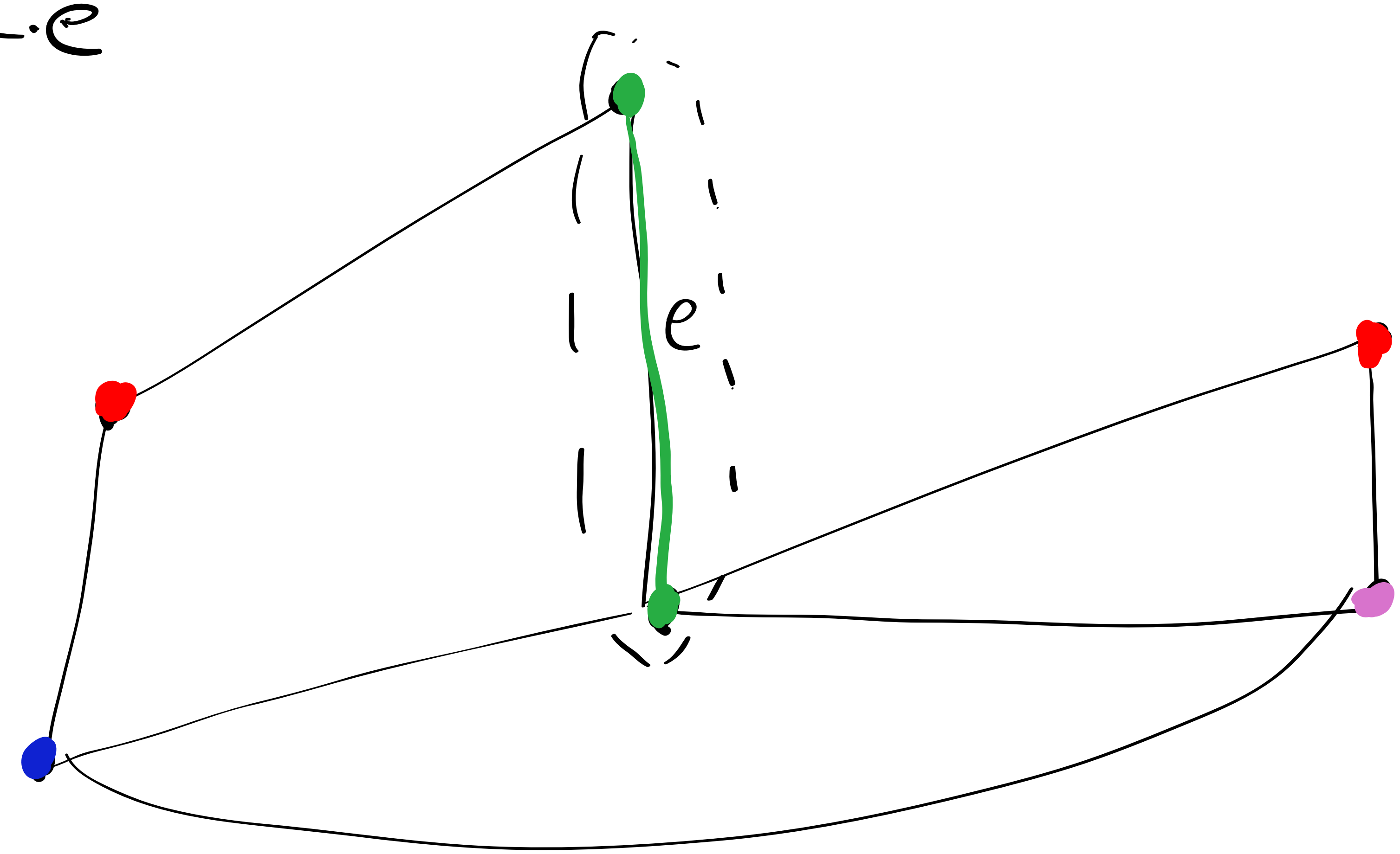
Theorem $P_G(n) = P_{G-\{e\}}(n) - P_{G\cdot e}(n)$

pf.



A coloring of the vertices of $G-\{e\}$ will color G (same vertices), as long as the two vertices incident to e are colored differently.

$G \cdot e$



A coloring of $G \cdot e$ corresponds exactly to a coloring of $G - \{e\}$ that is not valid for G.