Lecture 33. Planarity. Read all of §11.4.

Planarity Tests discussed there are impractical. The effort needed grows (I think) exponentially with the size of the graph.

Last time: If G is connected, simple and drawn in the plane: $n - m + f = 2$

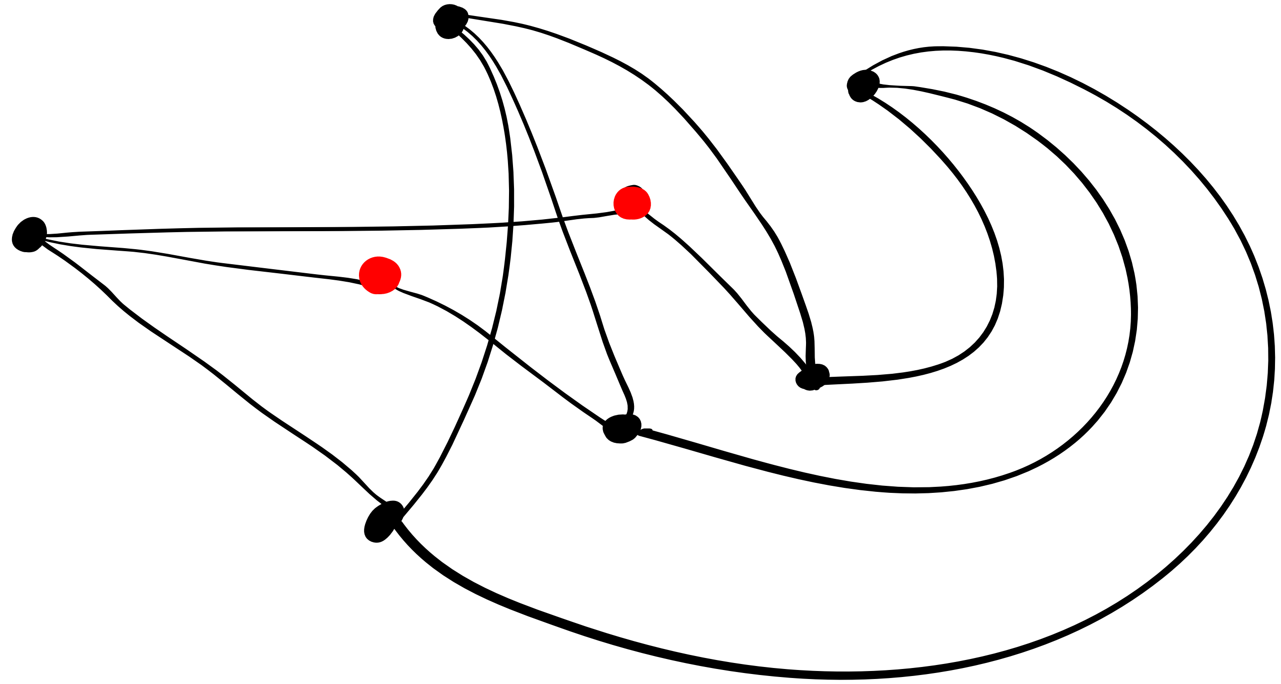
$\Longrightarrow$ • $m \leq 3n - 6$

$\Longrightarrow$ (• $m \leq 2n - 4$, if G is bipartite)

$\Longrightarrow$ • $\exists$ a vertex of degree ≤ 5.

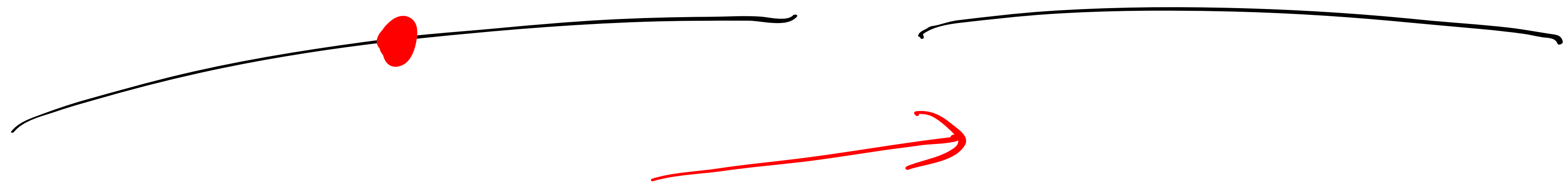
Today: 3 - constructions

- Subdivisions
- Contract an edge
- dual graphs

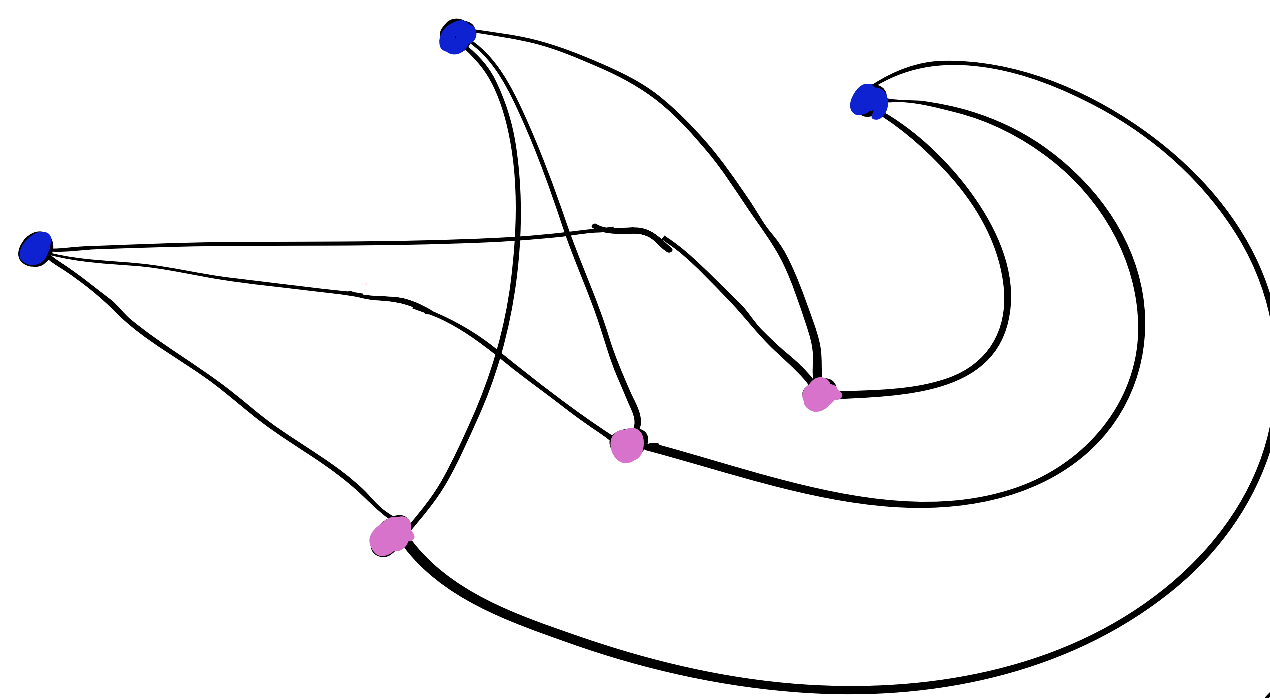


Not planar, since $K_{3,3}$ is not planar.

degree 2

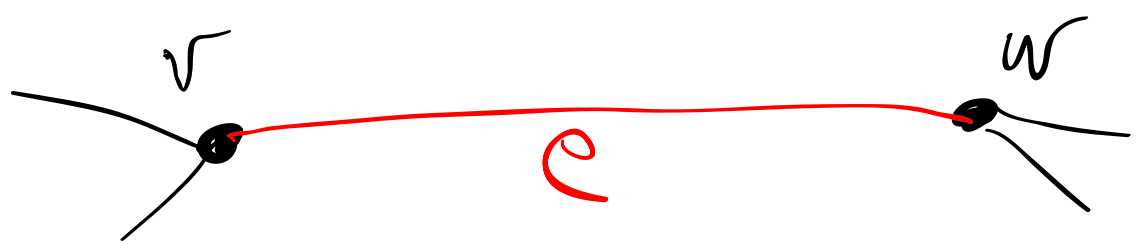


remove w/o altering planarity.

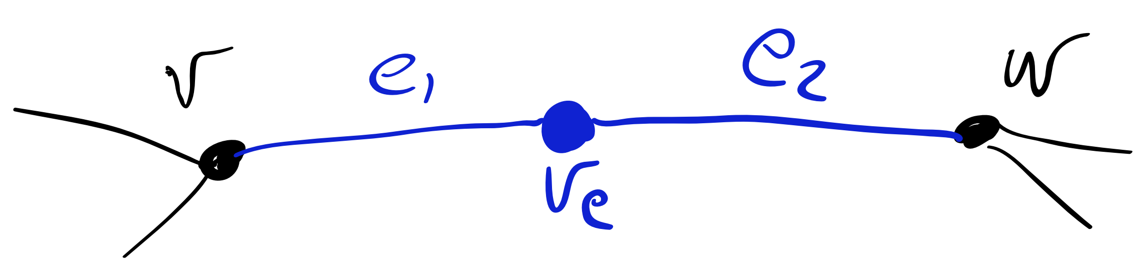


$K_{3,3}$

Given G, a subdivision of G is a graph derived from G by replacing e



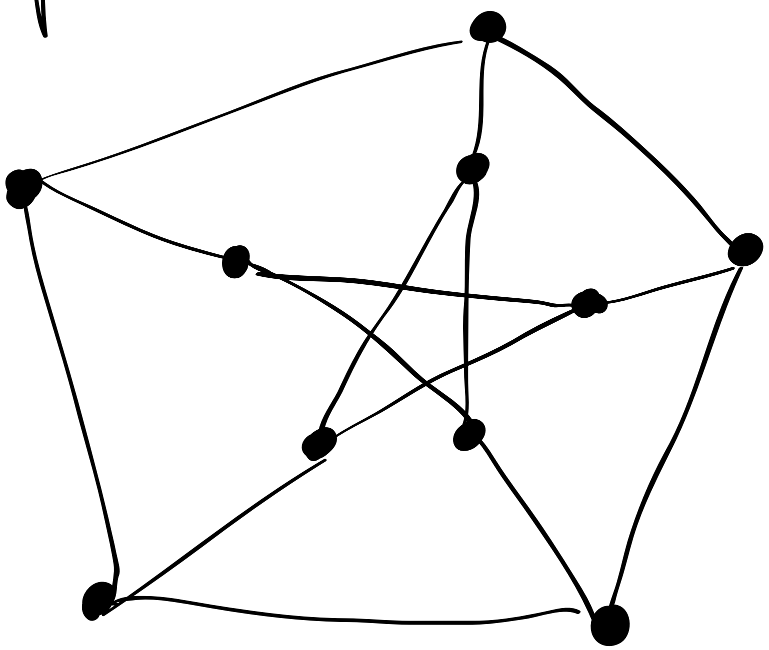
by e_1, e_2, and new vertex v_e as indicated:

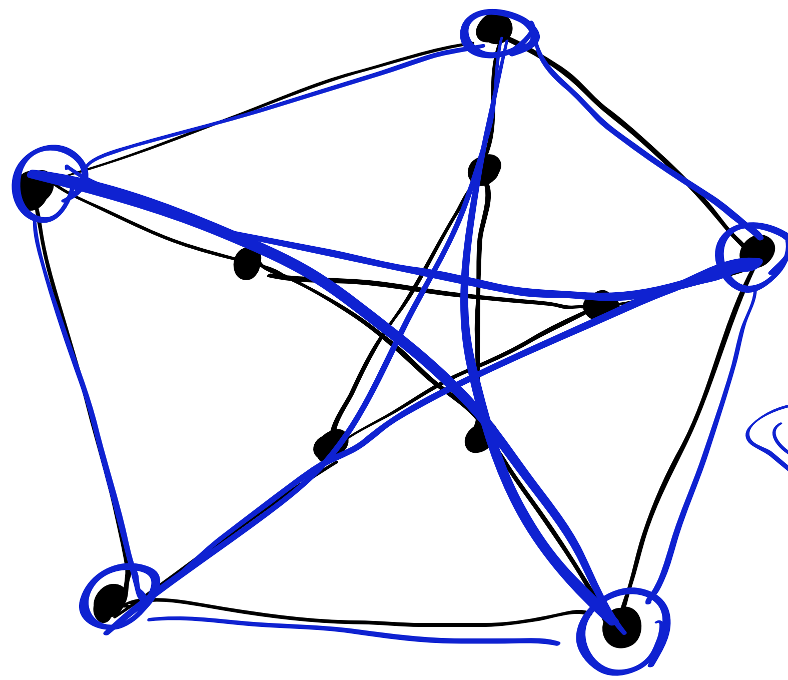


This process can be repeated and we still call the result a subdivision of G.

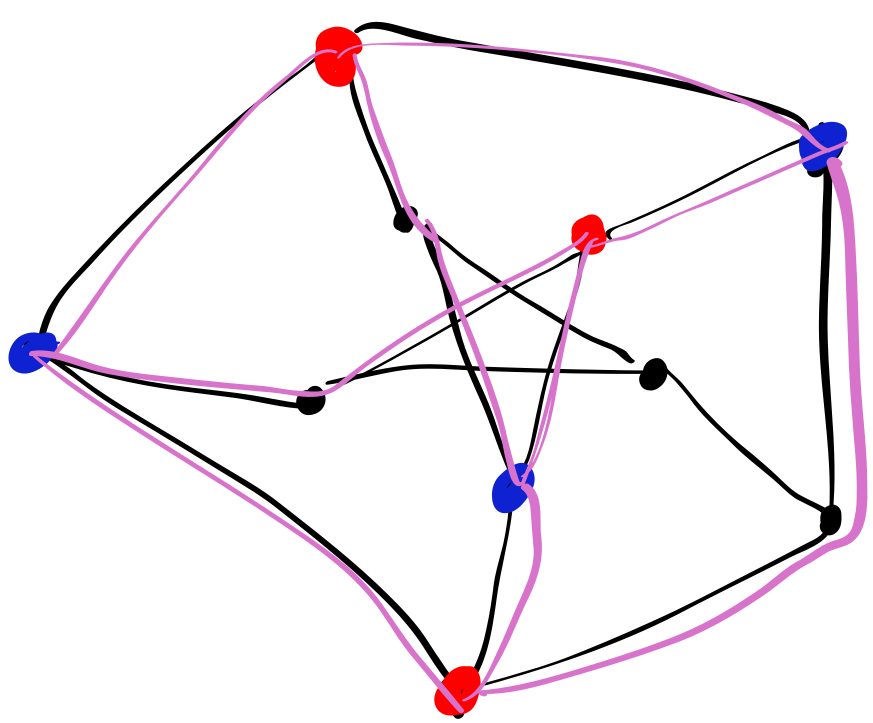
planar?

Look at subgraphs: K_5? No - degree of vertices is wrong.





What I drew in blue is not a subgraph. Oops.



$K_{3,3}$?

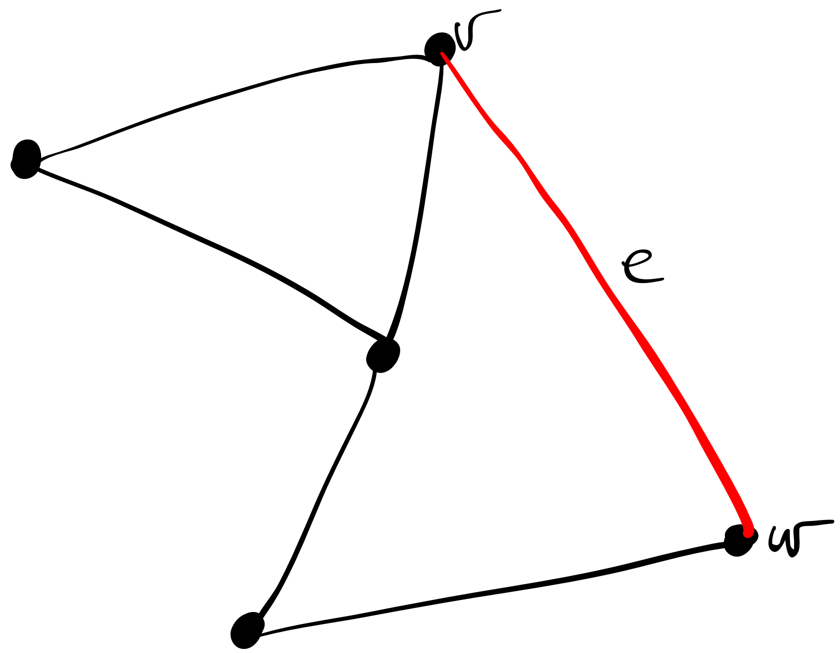
Blue & Purple shows a subdivision of $K_{3,3}$.

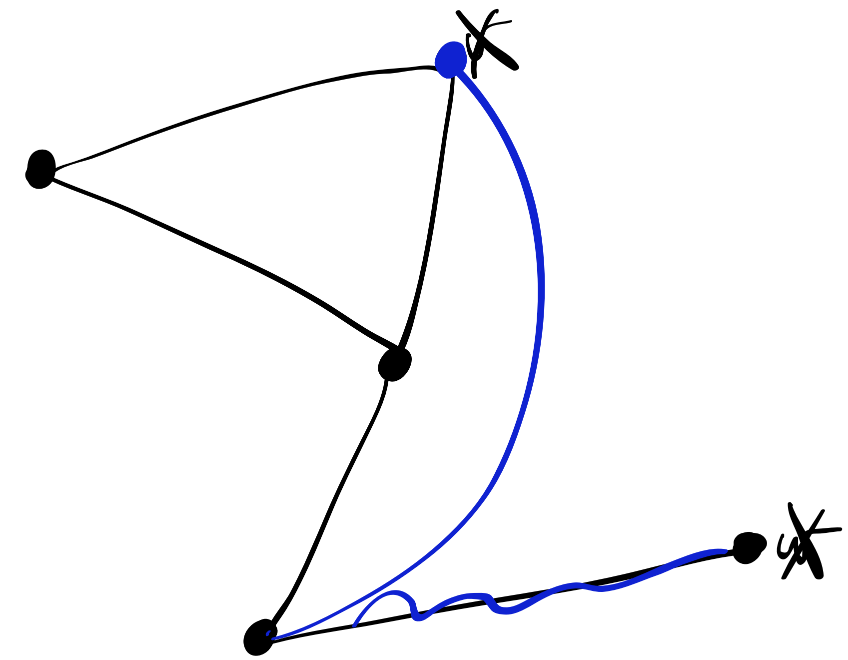
$\therefore$ Petersen graph is not planar

Theorem G, simple & connected, is planar iff it does not contain a subgraph isomorphic to a subdivision of $K_{3,3}$ or K_5.

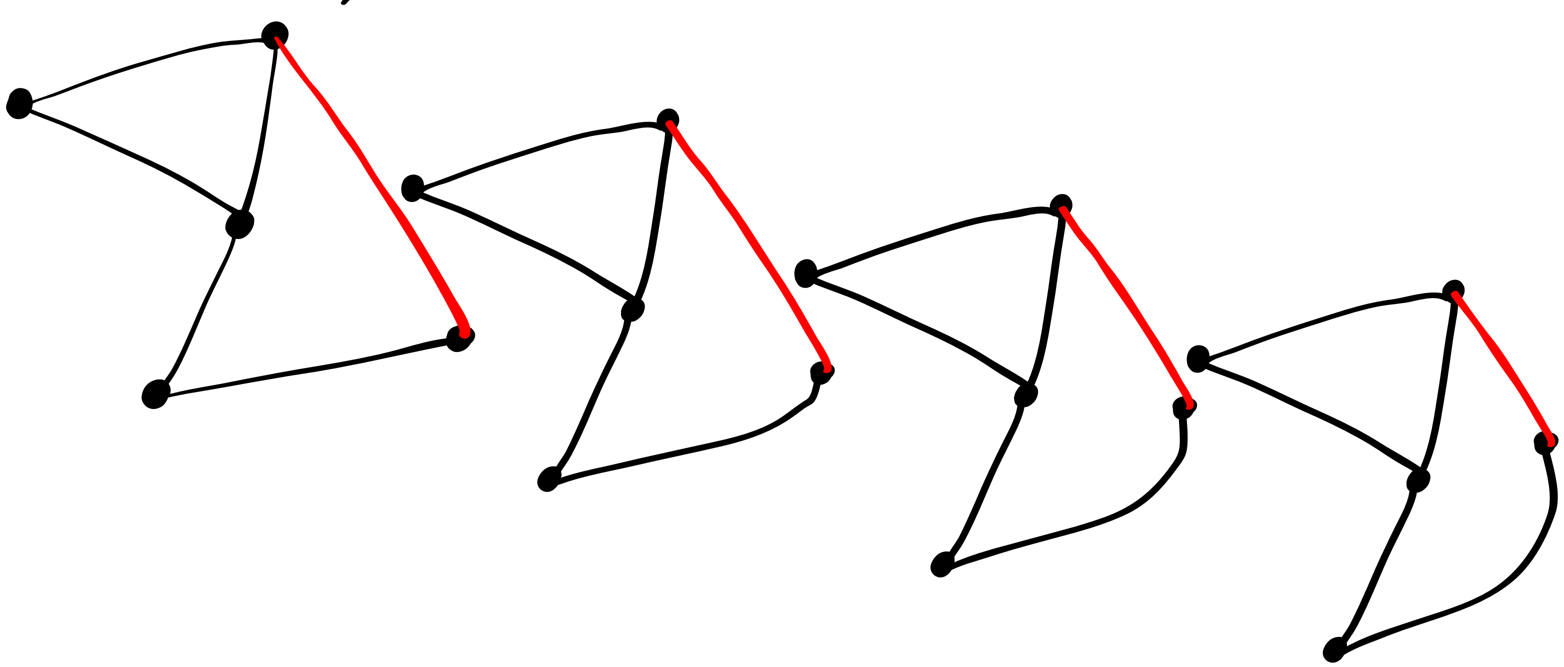
We're skipping the proof.

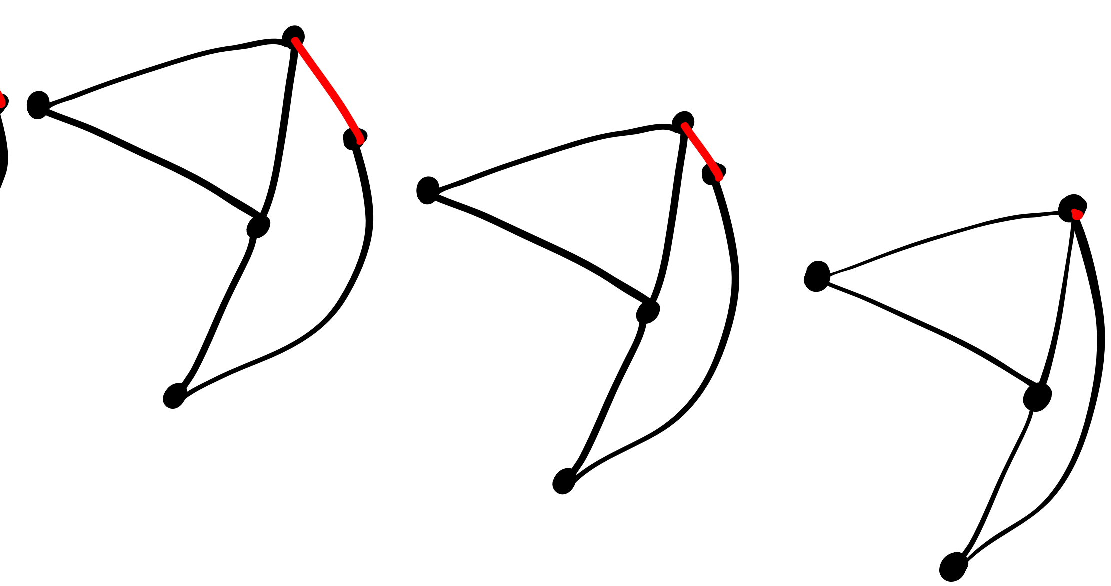
G.





Contracting G along e gives a new graph with one fewer vertex: (e ≠ loop)





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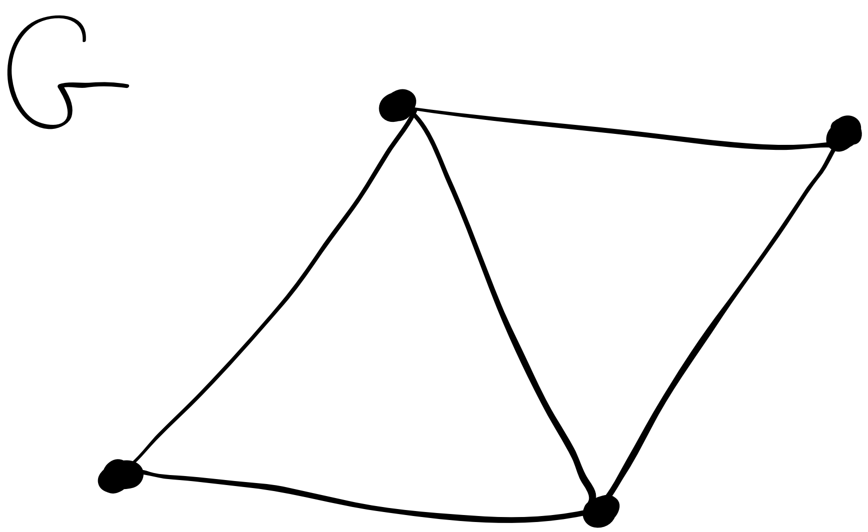
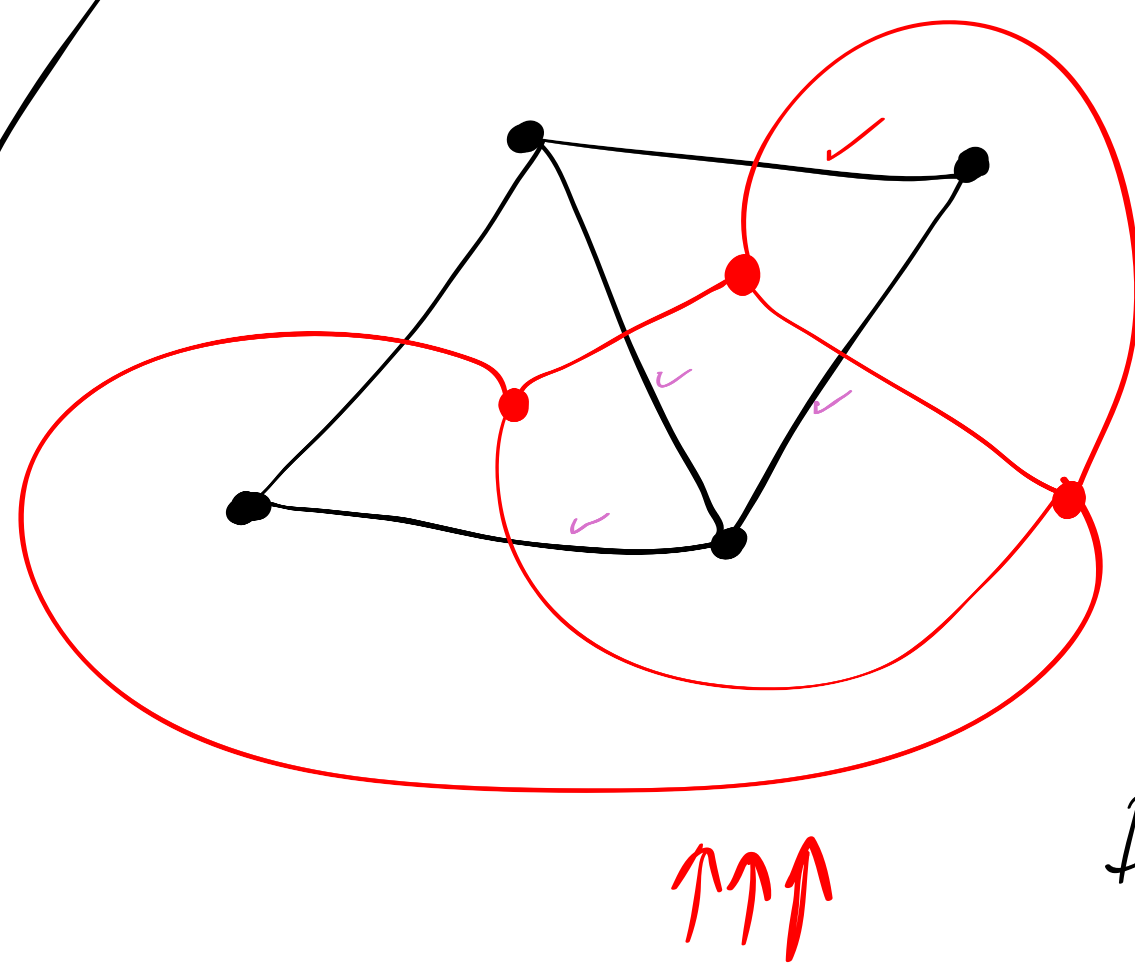
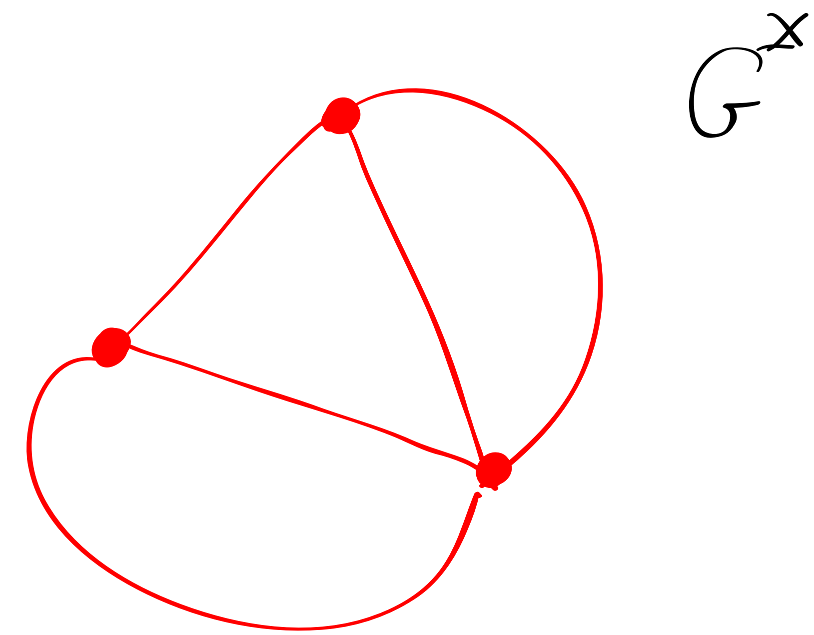
Dual Graphs. This is a concept that applies only to graphs drawn in the plane.

Swap vertices & faces.







4 vertices
5 edges
3 faces

3 vertice
5 edgess
4 faces

Dual of G^*? G!