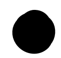
Lecture 31. Building graphs from "blocks".

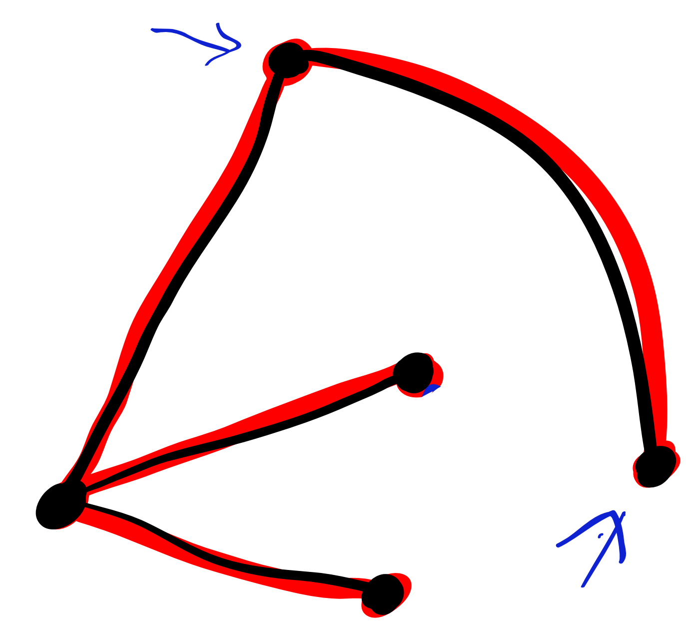
Let T be a tree. We'll build with

one of these: 

Lots of these: "needle" 

<u>Claim</u>: Can build a tree from these, following

1 Rule: Stick pointy end of needle in a vertex:



We've proven: for a tree with n vertices and m edges: $1-0=1$

Start: ● $\quad n-m=1$

Add ——● $\quad n \leftarrow n+1$, $m \leftarrow m+1$

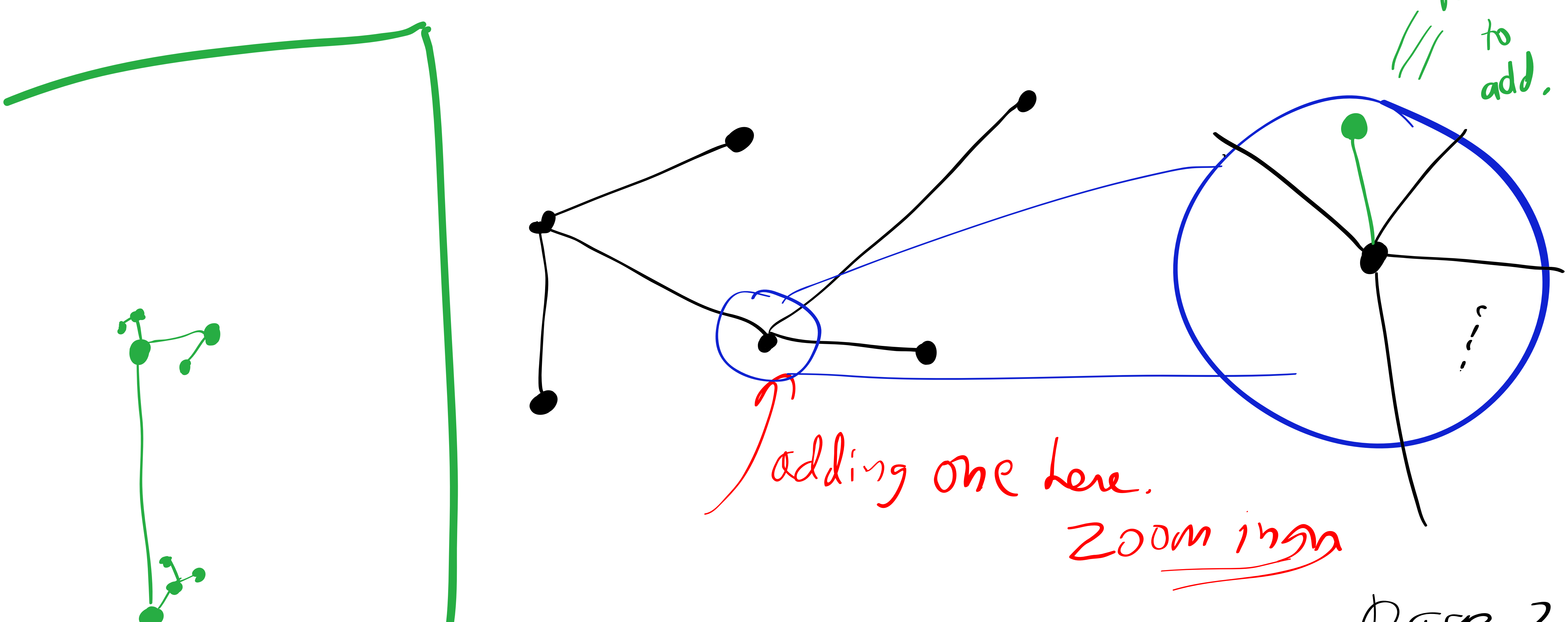
Still $n-m=1$

A tree has one more vertex than it has edges.

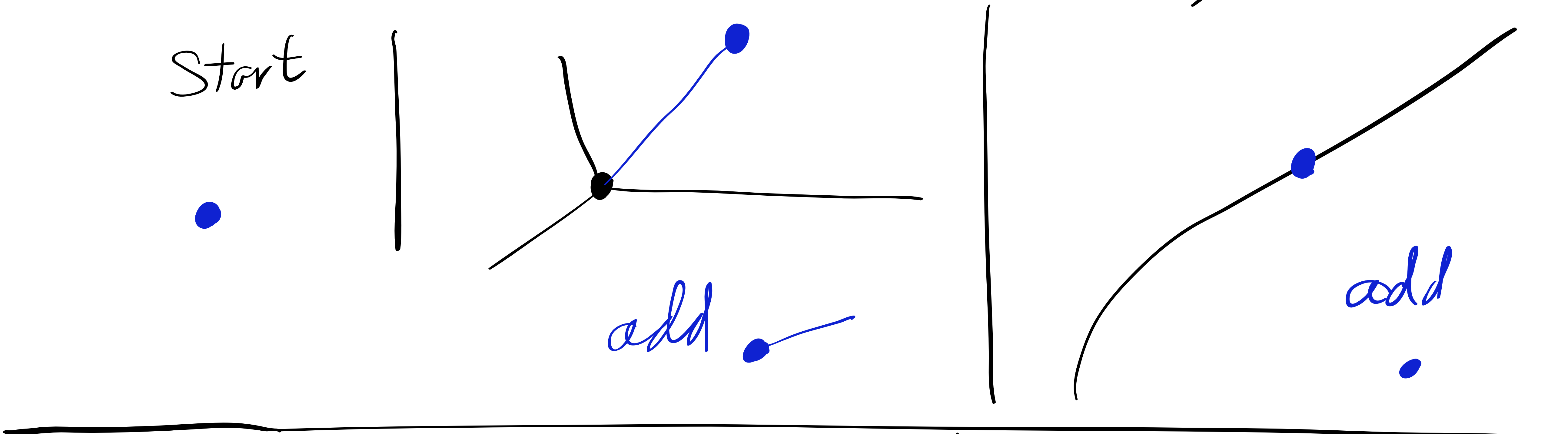
In a tree, there is always one more vertex than edges.

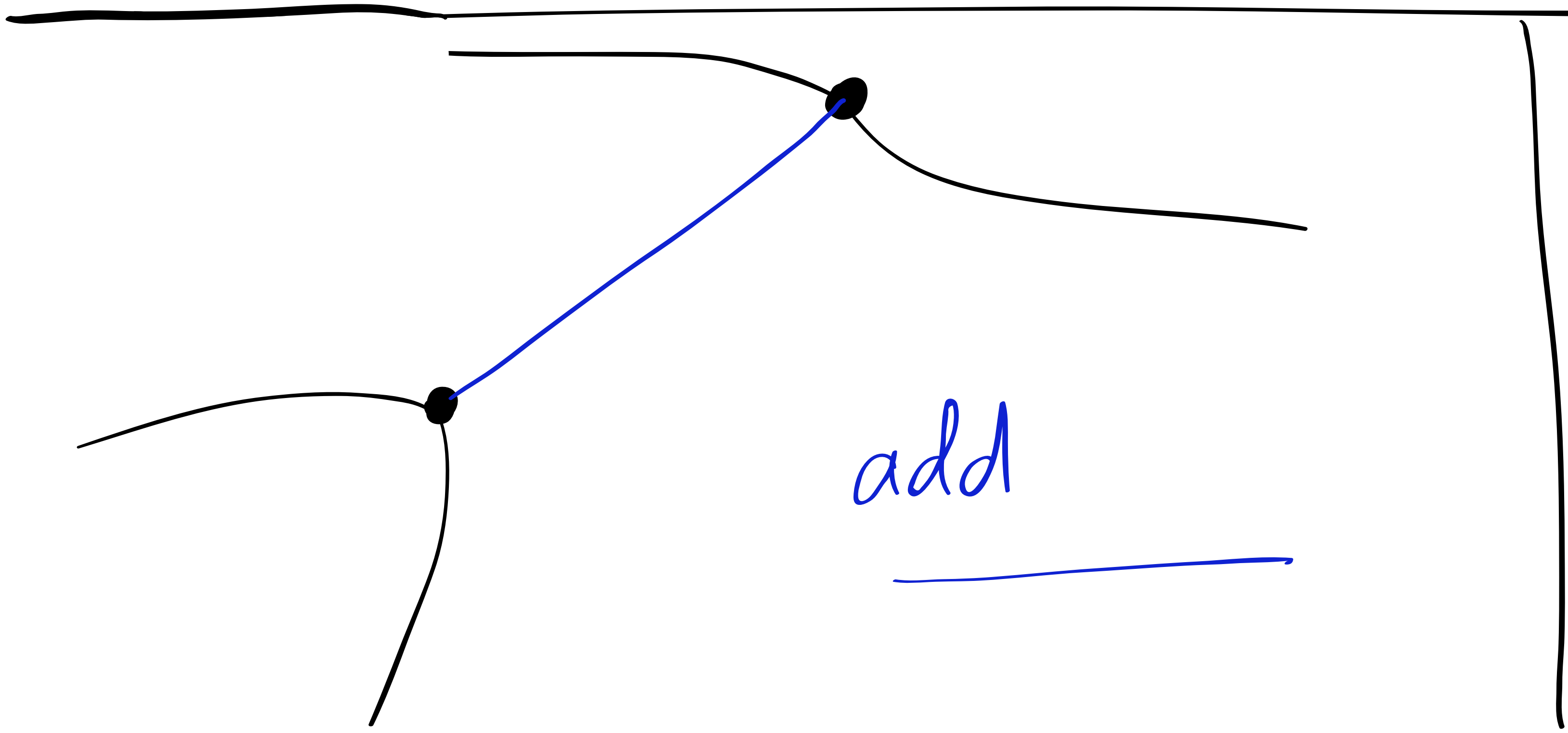
Proven: Trees are planar.

Also: drawing of trees satisfy Euler's formula.



Claim Any connected graph can be constructed from the following "moves":

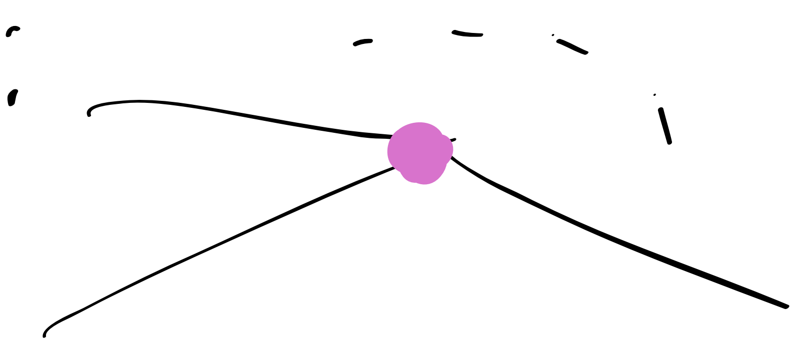




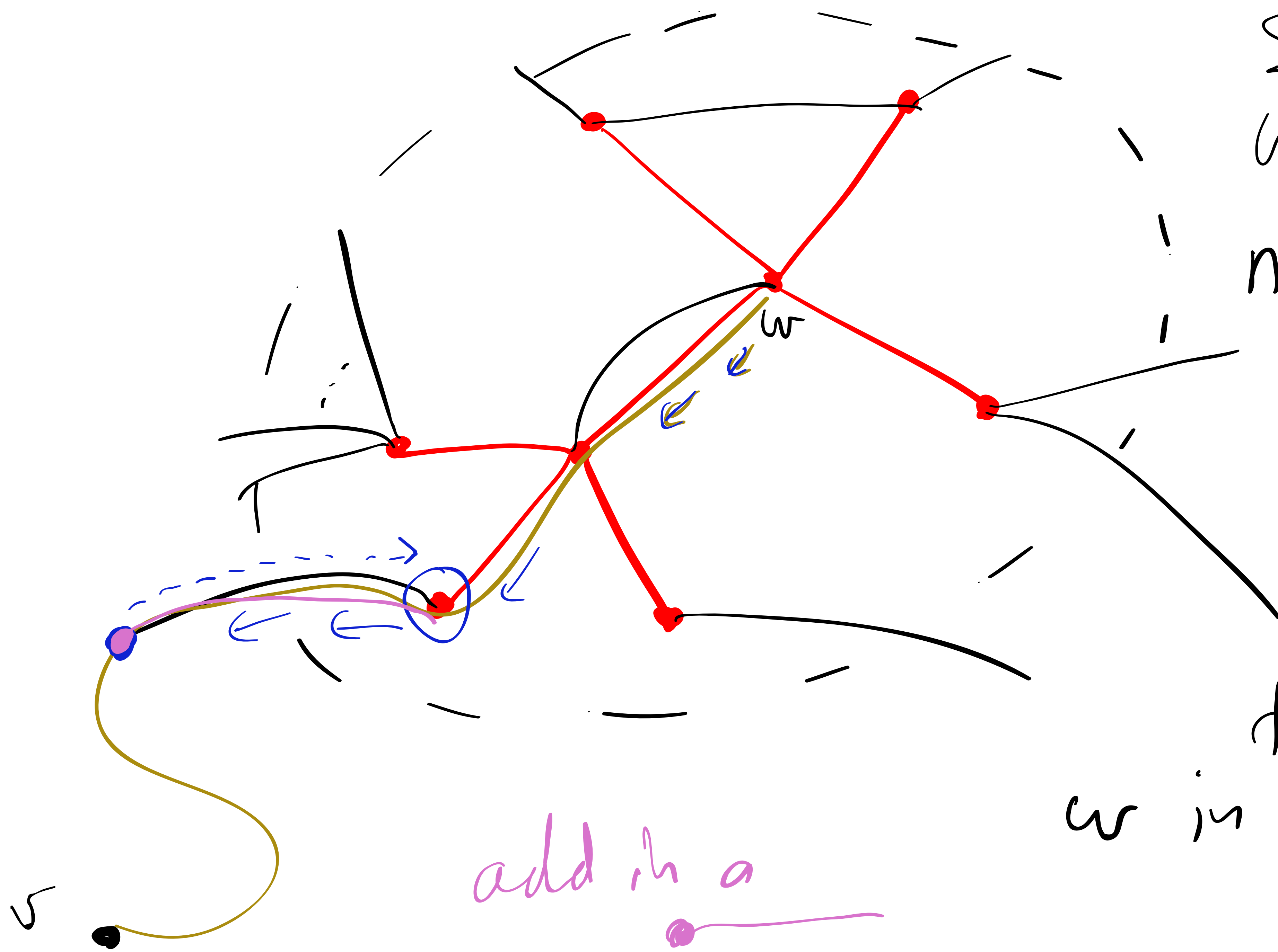
<u>Lemma</u> In every every connected graph G, with n vertices there is a tree (a subgraph that is a tree) with n vertices.

<u>pf:</u> <u>Lemma ++</u> For this G, if $1 \leq r \leq n$, there is a tree, a subgraph of G, with r vertices.

<u>pf.</u> Base case: <u>$r=1$</u>. In this case, we need a copy of $\bullet$ in G:



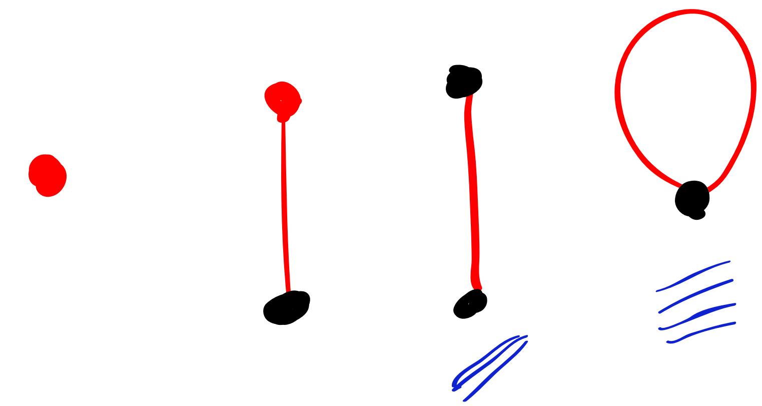
Assume G has a subgraph that is a tree with (r-1) vertices.



Since r-1 < n, We can pick v, not in the tree.

Find a path from v to any w in the tree.

□ □

Back to showing any G is built from the moves: 

We know $\exists\ T \subseteq G$, a tree.
T can be built from these moves.