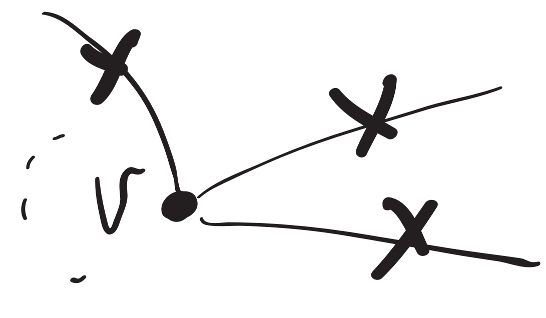
Lecture 28. Relations between measures of connectivity.

G a connected.

Theorem $\underbrace{\kappa(G) \le \lambda(G)}_{(2)} \underbrace{\le \delta(G)}_{(1)}$

(1) Pick on a vertex v of degree $\delta(G)$;

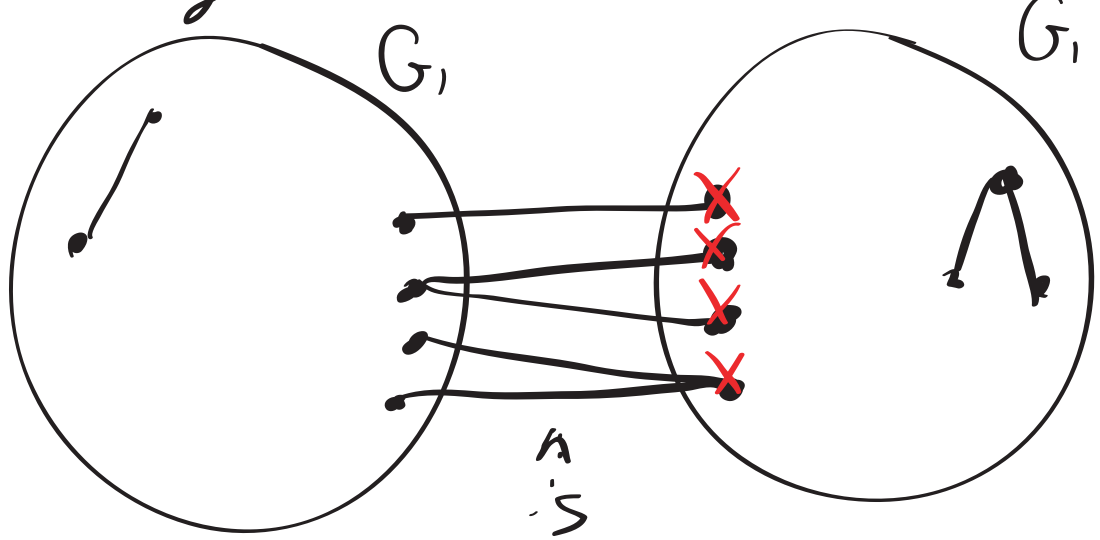


$G = K_1$ •

$\kappa(\bullet) = \lambda(\bullet) = \delta(\bullet) = 0$

Define as 0

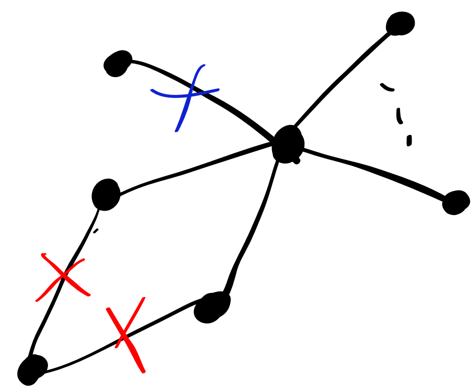
(2) Suppose $k = \lambda(G)$. Consider a set S of edges that disconnects G.



Book's argument is fine unless $G = K_n$.

$\delta(G)$ is <u>not</u> a good estimate on $\lambda(G)$;

G



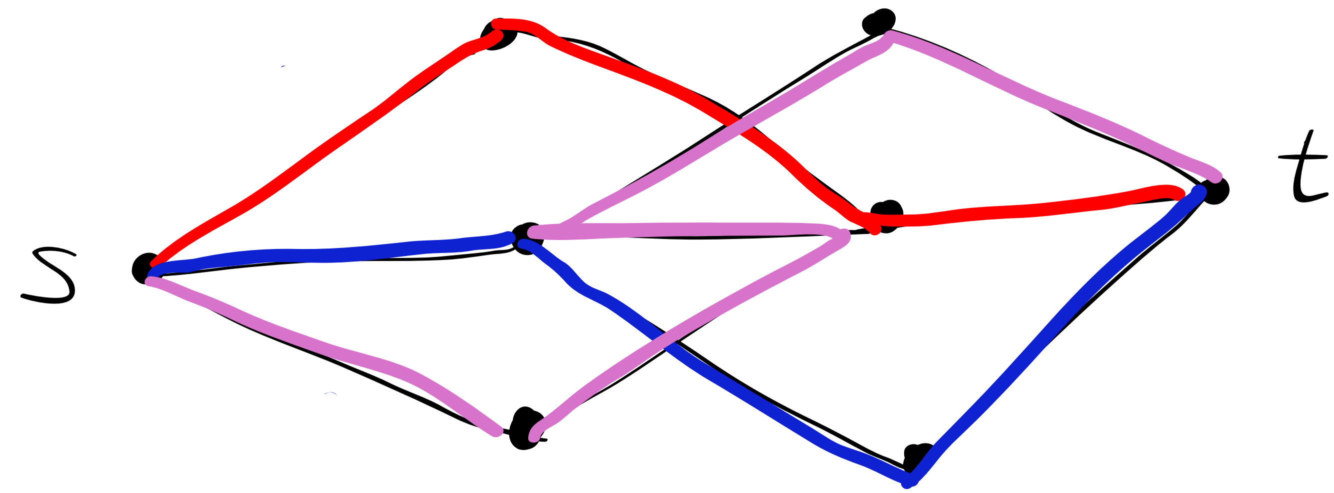
$\delta(G) = r$ (any $r \in \mathbb{N}$)

$\lambda(G) = \cancel{2}\ 1$

Menger's Theorem(s).

<u>Digraphs</u> & Graphs

easiest to understand is the arc-form on digraphs.



How many edges need to be removed to separate s from t. 3.

How many paths can we find from s to t so that no two share an edge?

Let D be a digraph. Suppose s and t are vertices.

Def: an st-path is any path from s to t.

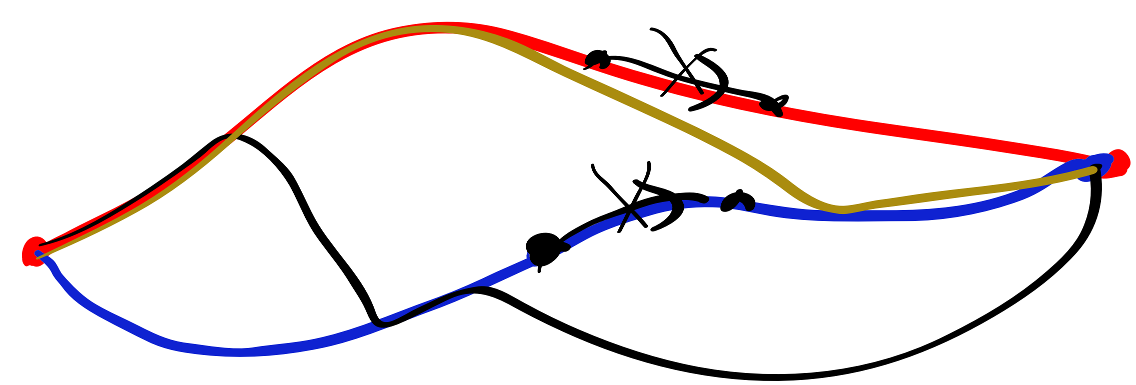
Def: Two st-paths are arc-disjoint if they share no arcs.

Def: A set $\{P_1, P_2, \ldots, P_n\}$ of st-paths is arc-disjoint if all pairs P_i, P_j are arc-disjoint, for $i \neq j$.

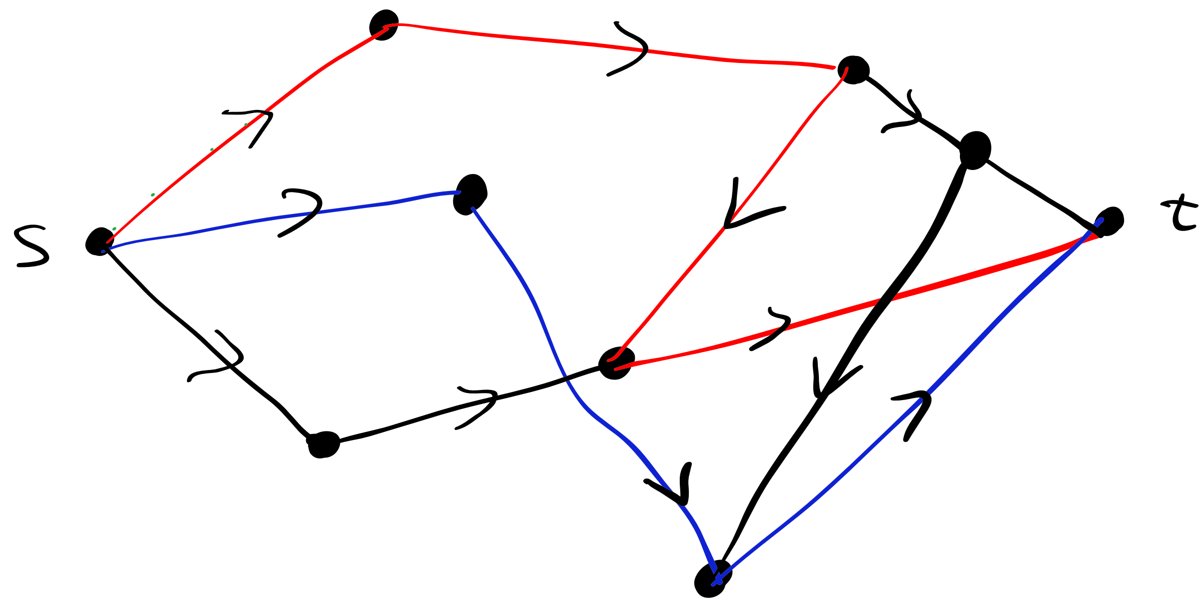
Relevant to communications.

Suppose A is a set of st-paths that are arc-disjoint. Let $k = |A|$.
How can we find k arcs that disconnect s and t?

Easy to find k arcs that destroy all of A. But $\exists$ more paths from s to t.



Here class got interesting. It seems the proof in the book of Menger's theorem is wrong, as the following example uncovered. What we are looking at is a digraph in which there are at most two disjoint paths from s to t. The red and blue paths form one such pair.



According to the proof in the book, we look at the set of vertices one. I've boxed these. The book says the arrows leaving the boxed vertices will be colored, which is fine, and that the arrows going into boxed vertices will not be colored. This is false, and I don't see how this proof can be fixed.