Lecture 27

G a connected graph –

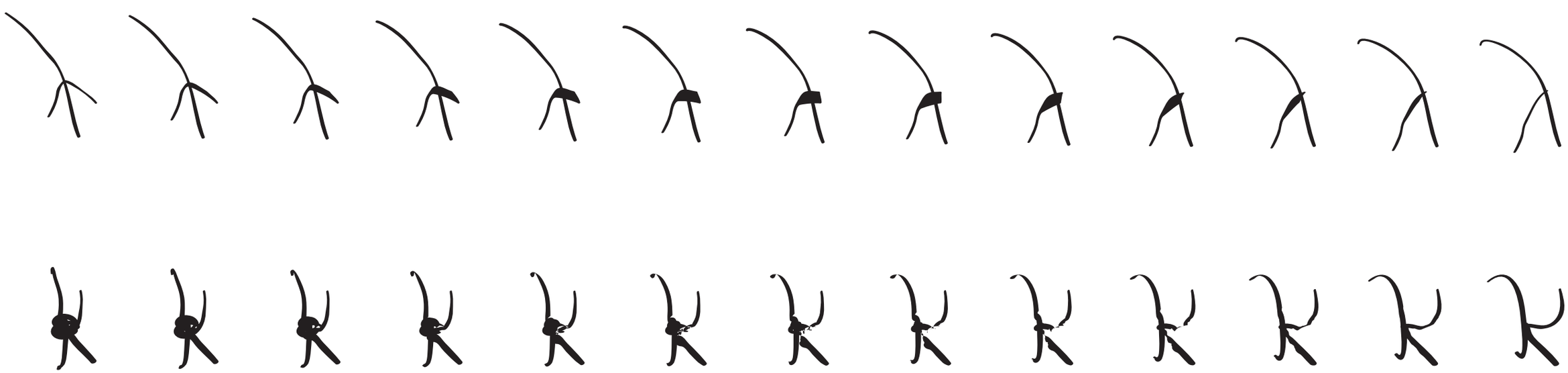
today we'll define three numbers associated to G:

$\kappa(G)$ – a measure vertex connectedness.

$\lambda(G)$ – a measure of edge connectedness.

$\delta(G)$ – max degree of a vertex.

How I remember which is which:



$\lambda(G)$ is the minimum number of edges that one can remove from G to disconnect it.

<u>Ex</u>. If G has a bridge, $\lambda(G)=1$.

Q: Can $\lambda(G)=0$? No, as G is connected.

Q: What is $\lambda(G)$ for $G = \bullet$

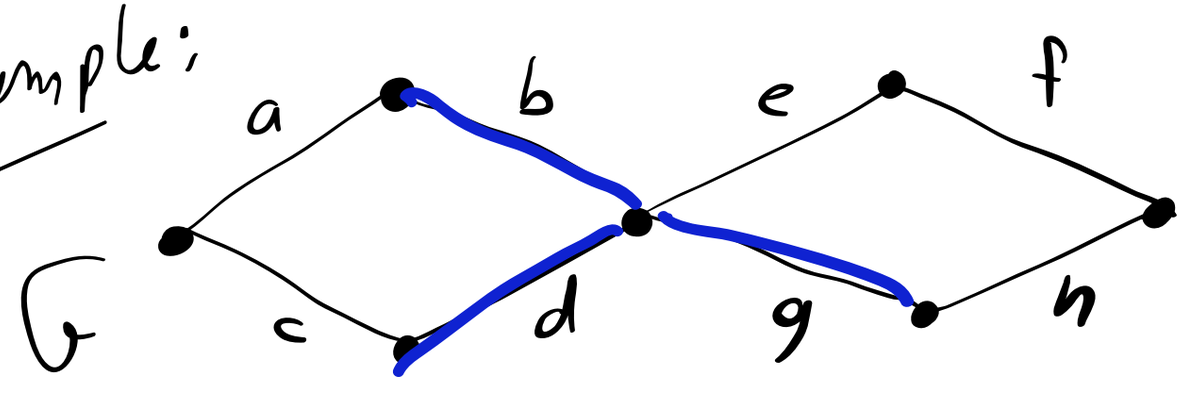
Will <u>Define</u> this as a special case to make theorems work.

($\lambda(\bullet)=0$ seems right.)

Def: If G is a connected graph, a **cut-set** of G is a set $S \subseteq E(G)$ s.t.

- Removing S creates a disconnected graph; and
- If $S_0 \subsetneq S$, then removing S_0 does not disconnect G.

Example:



$\lambda(G) = 2$

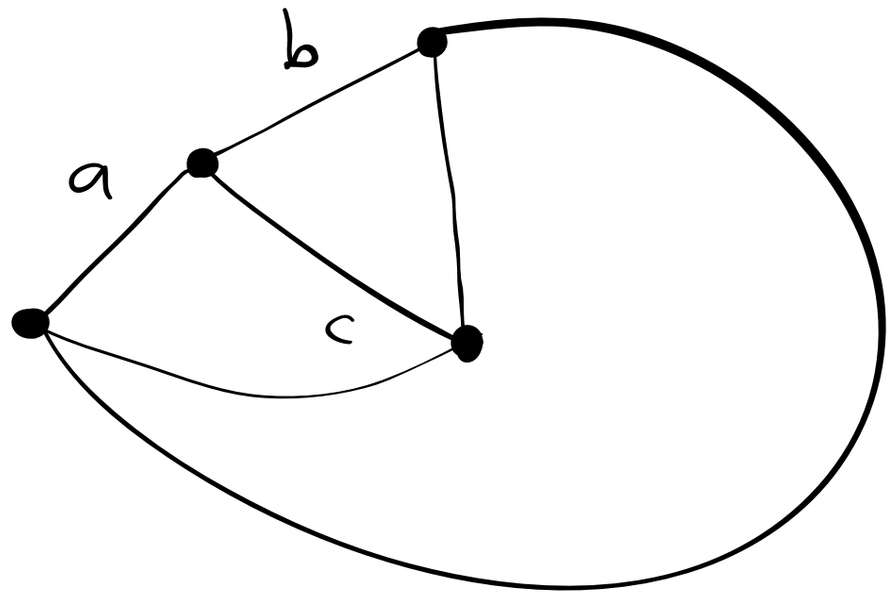
Some cut sets for G:

$\{a, b\}$, $\{a, c\}$

$\{b, g, d\}$ - disconnects.

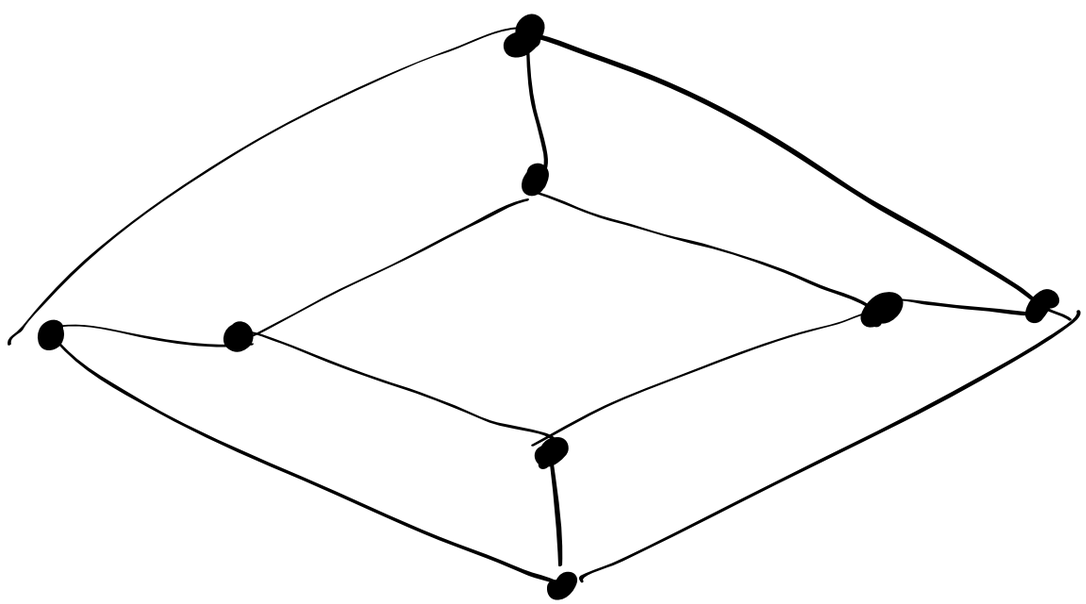
$\{b, d\}$ - also disconnects.

$\Rightarrow$ not a cut-set

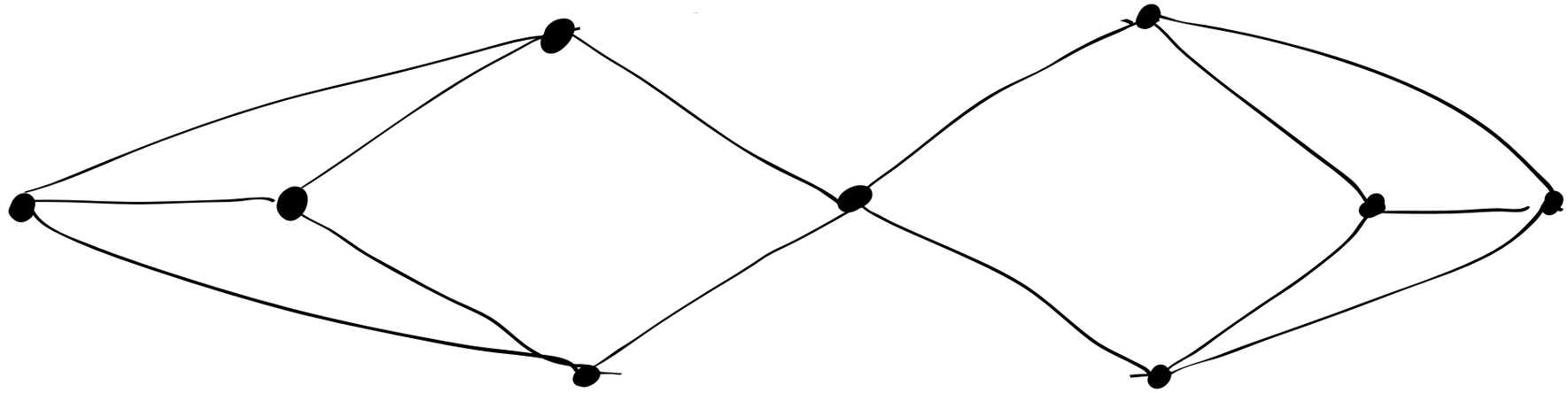


K_4

$\lambda(G) = 3$



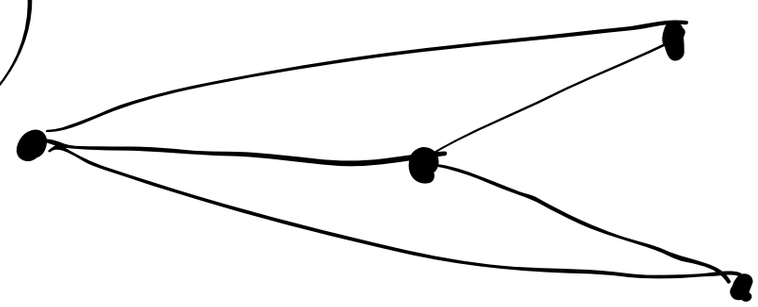
$\lambda(G) = 3$



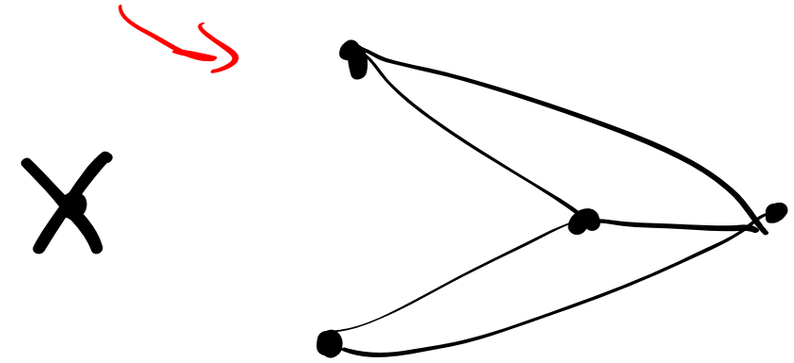
Def:

A **vertex-cut-set** is a set of vertices such that removing the set disconnects G, but removing any proper subset does not disconnect G.

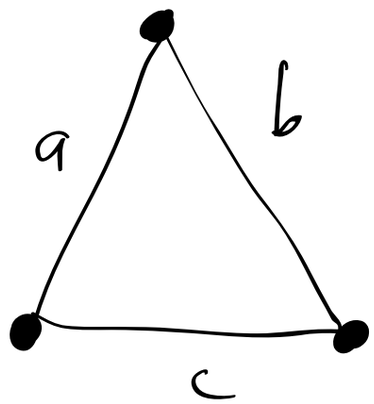
X means remove the vertex and all incident edges.



not connected

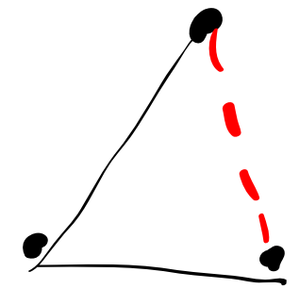


The minimum size of a vertex-cut-set is, by defn, $\kappa(G)$.



G

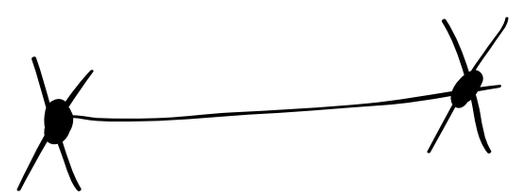
H



K

$\kappa(K) = 1$

Q: What are the <u>vertex-cut</u> sets of H?



$\emptyset$ (not a graph)

Ans: There are none.

<u>Def'n (special case):</u> $\kappa(K_n) = n-1$.

HW. § 9.1. 2(a)(c), 5(a)(d),
7(a)(c), 10(a)(b).

Due Friday, April 12