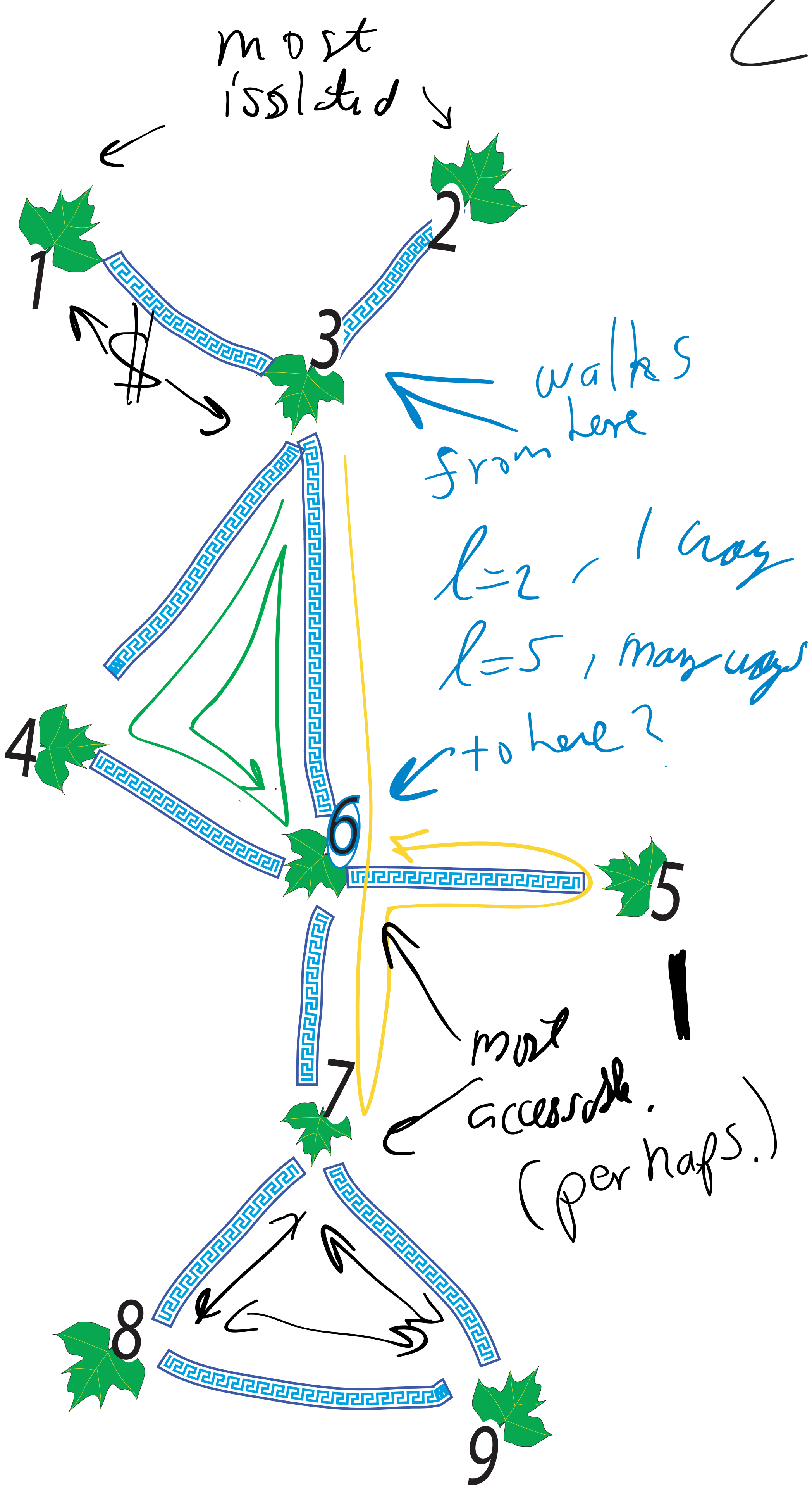
Lecture 26

Accessability: What is it, and how do we quantify it?



$$A = \begin{bmatrix} 0&0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 1&1&0&1&0&1&0&0&0\\ 0&0&1&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&1&1&1&0&1&0&0\\ 0&0&0&0&0&1&0&1&1\\ 0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&1&1&0 \end{bmatrix}$$

We calculate number of walks from vertex 6 to all vertices, of length 1, 2, ..., 20, ...: $x, Ax, A^2x, \ldots$ does this. Here I used MATLAB.

```
x =

     0
     0
     0
     0
     0
     1      <- #6
     0
     0
     0
```

```
>> x = (1/sum(A*x))*(A*x)      ==

x =

         0
         0
    0.2500
    0.2500
    0.2500
         0
    0.2500
         0
         0
```

```
>> x = (1/sum(A*x))*(A*x)

x =

    0.1000
    0.1000
    0.1000
    0.1000
         0      <- 6 -> 5 -> 6, etc.
    0.4000
         0
    0.1000
    0.1000
```

```
>>  .
    .
    .

    21 steps skipped
    .
    .
    .
```

```
>> x = (1/sum(A*x))*(A*x)

x =

    0.0650
    0.0650
    0.1735
    0.1379
    0.0729
    0.1937      <- approx equal.
    0.1327
    0.0796
    0.0796
```

```
>> x = (1/sum(A*x))*(A*x)

x =

  [ 0.0651
  [ 0.0651     more isolated from 6
    0.1733
    0.1378
    0.0727      <- a bit more accessible.
    0.1940
    0.1325
    0.0797
    0.0797
```

We do this again, But start at vertex 3.
The (1/sum(A*X)) is MATLAB code to scale down the vectors so we see relative numbers of walks.

```
x =

     0
     0
     1
     0
     0
     0
     0
     0
     0
```

← From vertex 3

```
>> x = (1/sum(A*x))*(A*x)

x =

    0.2500
    0.2500
         0
    0.2500
         0
    0.2500
         0
         0
         0
```

deg(3)=4

```
>> x = (1/sum(A*x))*(A*x)

x =

         0
         0
    0.5000
    0.1250
    0.1250
    0.1250
    0.1250
         0
         0
```

$l = 2$

```
>> .
 .
 .

 24 steps skipped
 .
 .
 .
```

```
>> x = (1/sum(A*x))*(A*x)

x =

    0.0652
    0.0652
    0.1732
    0.1378
    0.0726
    0.1942
    0.1324
    0.0797
    0.0797
```

$l = 26$

approx =

```
>> x = (1/sum(A*x))*(A*x)

x =

    0.0650
    0.0650
    0.1736
    0.1379
    0.0729
    0.1937
    0.1327
    0.0796
    0.0796
```

This vector always shows up.

6.5% access of 1 to 3.

A^{26} – the i-j entry tells the # of walks of length 26 from i to j.

$x = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \leftarrow k^{\text{th}}$ spot $\qquad A^{26}x = k^{\text{th}}$ column of A^{26}.

So examine: x, Ax, A^2x, A^3x, …

Easier: $A^3x = A(A(Ax))$ // $\Leftarrow$ size of entries grows like 2^n!

Instead: $x/\text{sum}(x)$, $Ax/\text{sum}(Ax)$ ⋯ ($\text{sum}(v)$ = sum of entries.)

Assume $\text{sum}(x) = 1$. $x_2 = \frac{1}{\text{sum}(Ax)} \cdot Ax$, $\quad x_3 = \frac{1}{\text{sum}(Ax_2)} \cdot Ax_2$ …

If you had linear algebra...

The vector is "accessibility" is an eigen-vector of A.

How do you find the eigenvalues of 9x9 matrix?

calculate (not fun) a degree 9 polynomial. Factor it, ...

This is why, in practice, we calculate

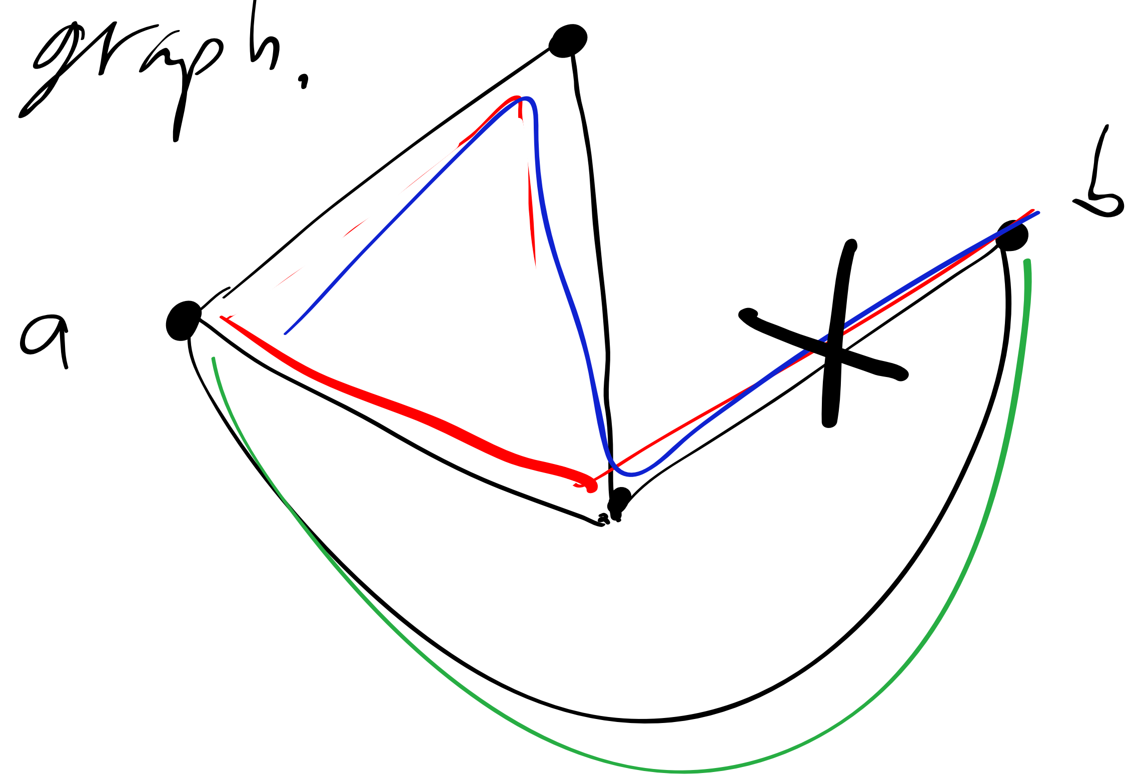
x, $(\text{sum}(Ax))^{-1} Ax$, ...

Also can start with $x = \begin{bmatrix} 1/9 \\ 1/9 \\ \vdots \\ 1/9 \end{bmatrix}$

$\frac{1}{\text{sum}(Ax)} Ax, \ldots$ get same result!

(Why? take a class in numerical analysis or applied matrix theory.)

Finding the "weakest link" (or set of links) in a graph.



3 routes (a to b). But one edge removed ruins 2 of the routes

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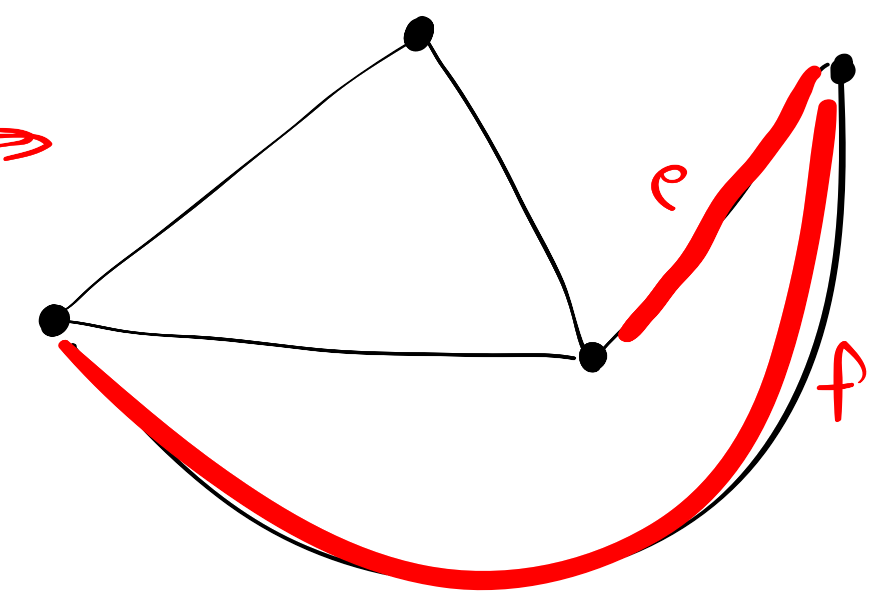
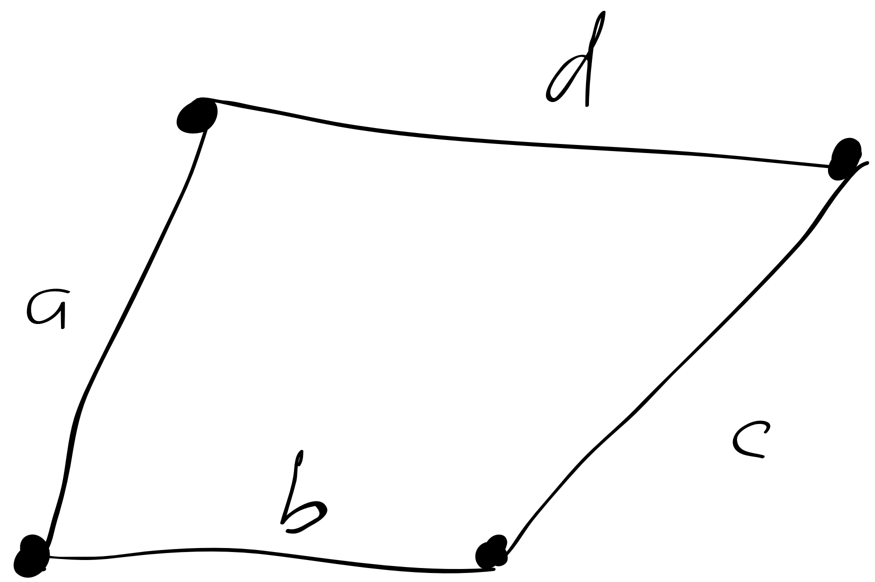
If you Remove e and f, you disconnect the graph. Removing only e or only f, does not disconnect the graph. (Remove e, f becomes a bridge.)



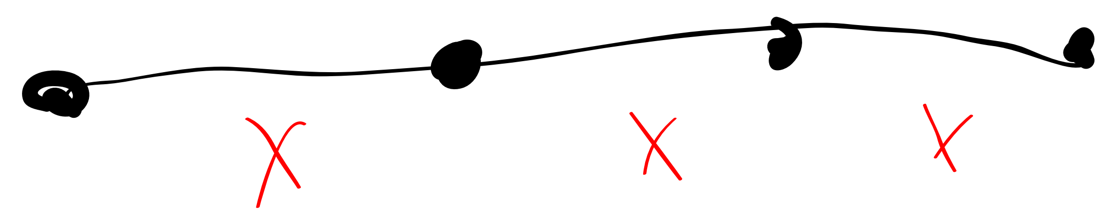
Ex: {e} is a cut-set if e is a bridge.

G



no brides.

Remove 1 edge, get



every 2 element set of edges is a cut-set, for this G.

remove one - disconnect remainig graph.