Lecture 23

Matrix Multiplication.

$$\begin{bmatrix} \cancel{x} & \cancel{x} & \cancel{x} \\ a & b & c \\ \cancel{x} & \cancel{x} & \cancel{x} \end{bmatrix} \cdot \begin{bmatrix} \cancel{x} & \cancel{x} & x \\ \cancel{x} & \cancel{x} & y \\ \cancel{x} & \cancel{x} & z \end{bmatrix} = \begin{bmatrix} & & \\ & & \boxed{?} \\ & & \end{bmatrix}$$

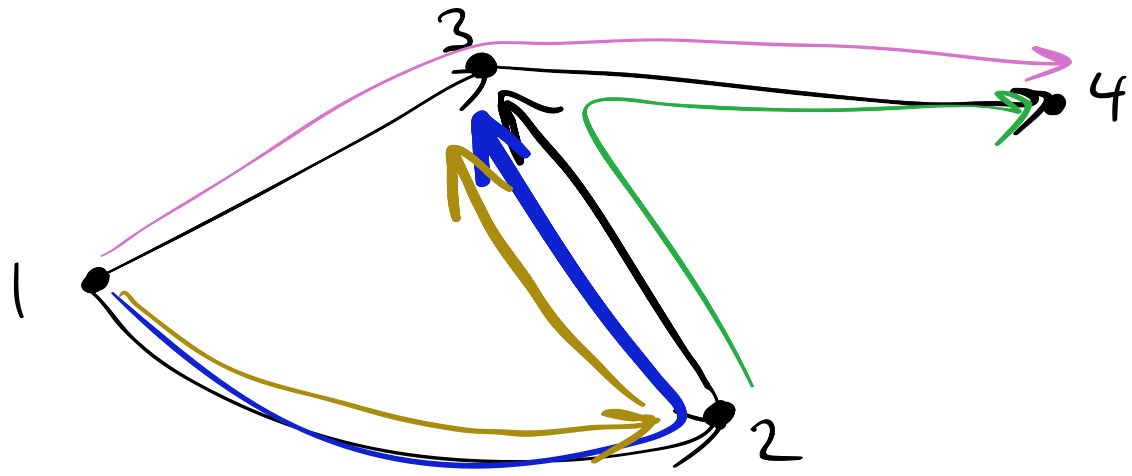
$? = ax + by + cz$

$$\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 8 & 3 \end{bmatrix}$$

$1 \cdot 4 + 0 \cdot 2 = 4$ | $1 \cdot 4 + 2 \cdot 2 = 8$

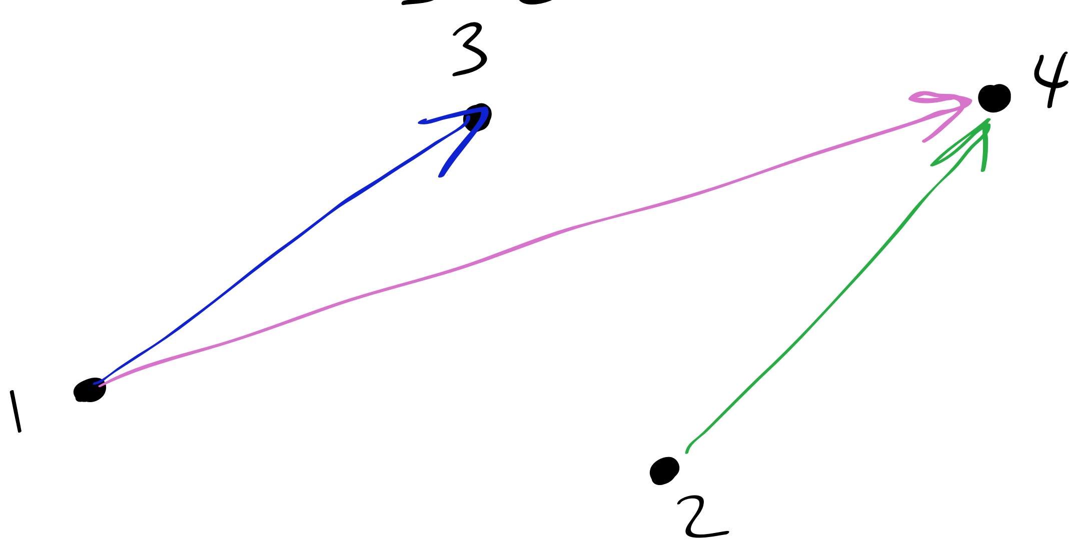
$1 \cdot (-1) + 0 \cdot 2 = -1$ | $1 \cdot (-1) + 2 \cdot 2 = 3$

$AB \neq BA$

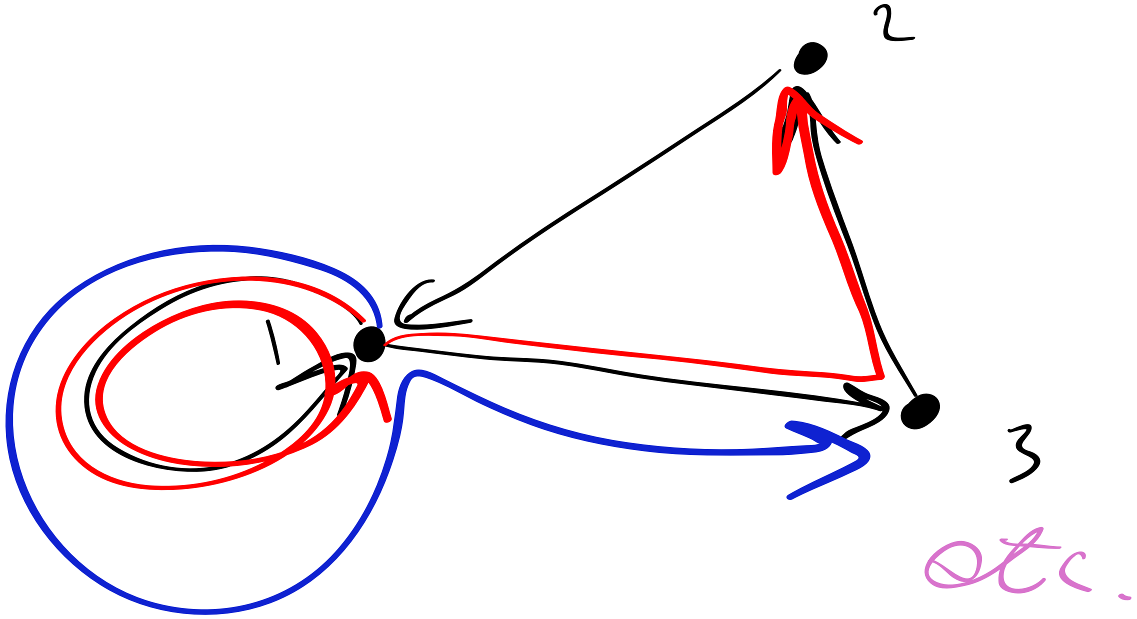


$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{array}{c} \text{From} \backslash \text{To} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



A^2 is adj matrix where each arc reps. a walk of length 2 in original graph.



$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

$$\boxed{I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

If D has n vertices, compute

$$C = I + A + A^2 + \cdots + A^n.$$

If C has no zero elements, D is connected.

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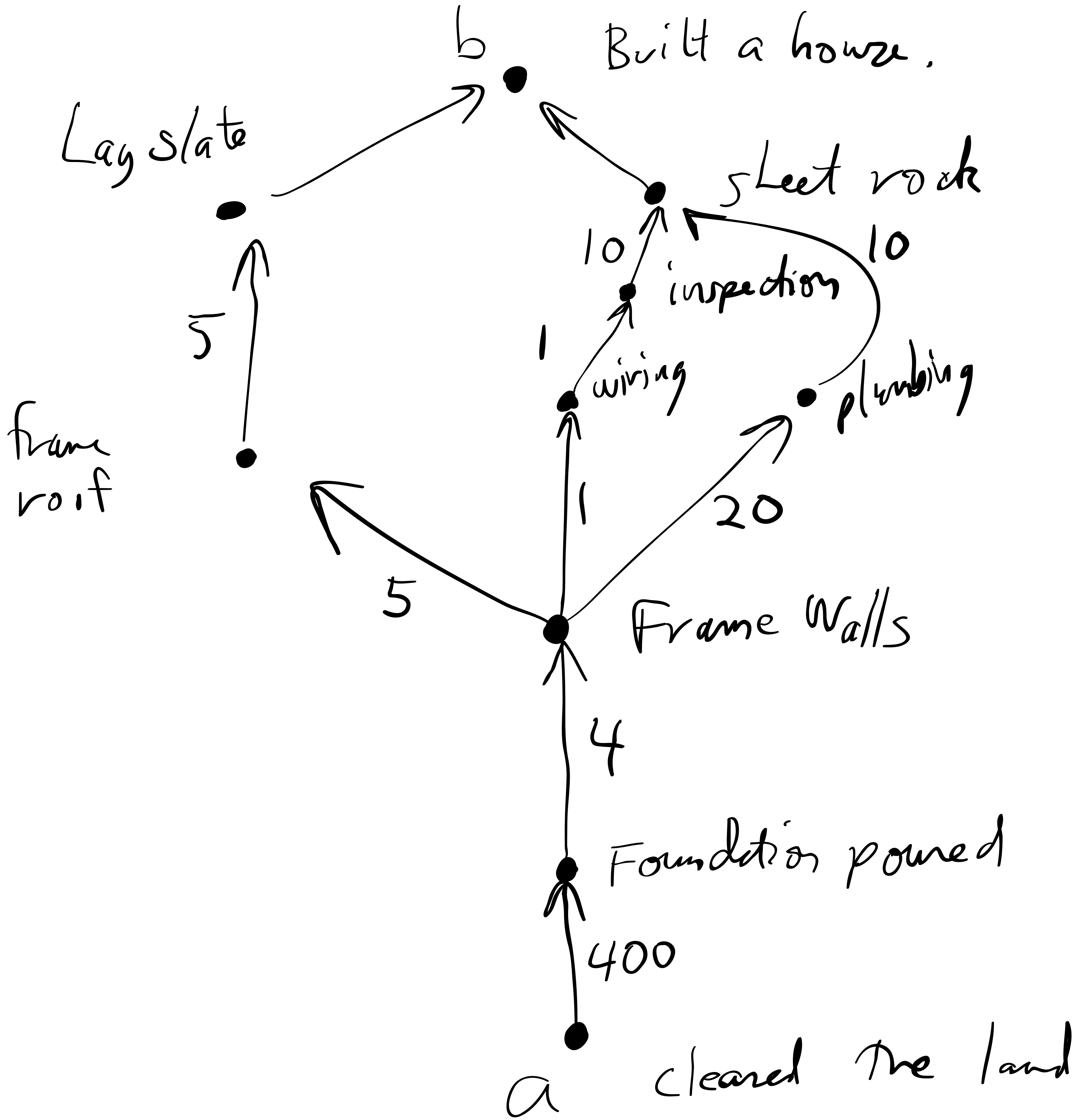
Can't use $A, A^2, \ldots,$

(at least easily) to solve:

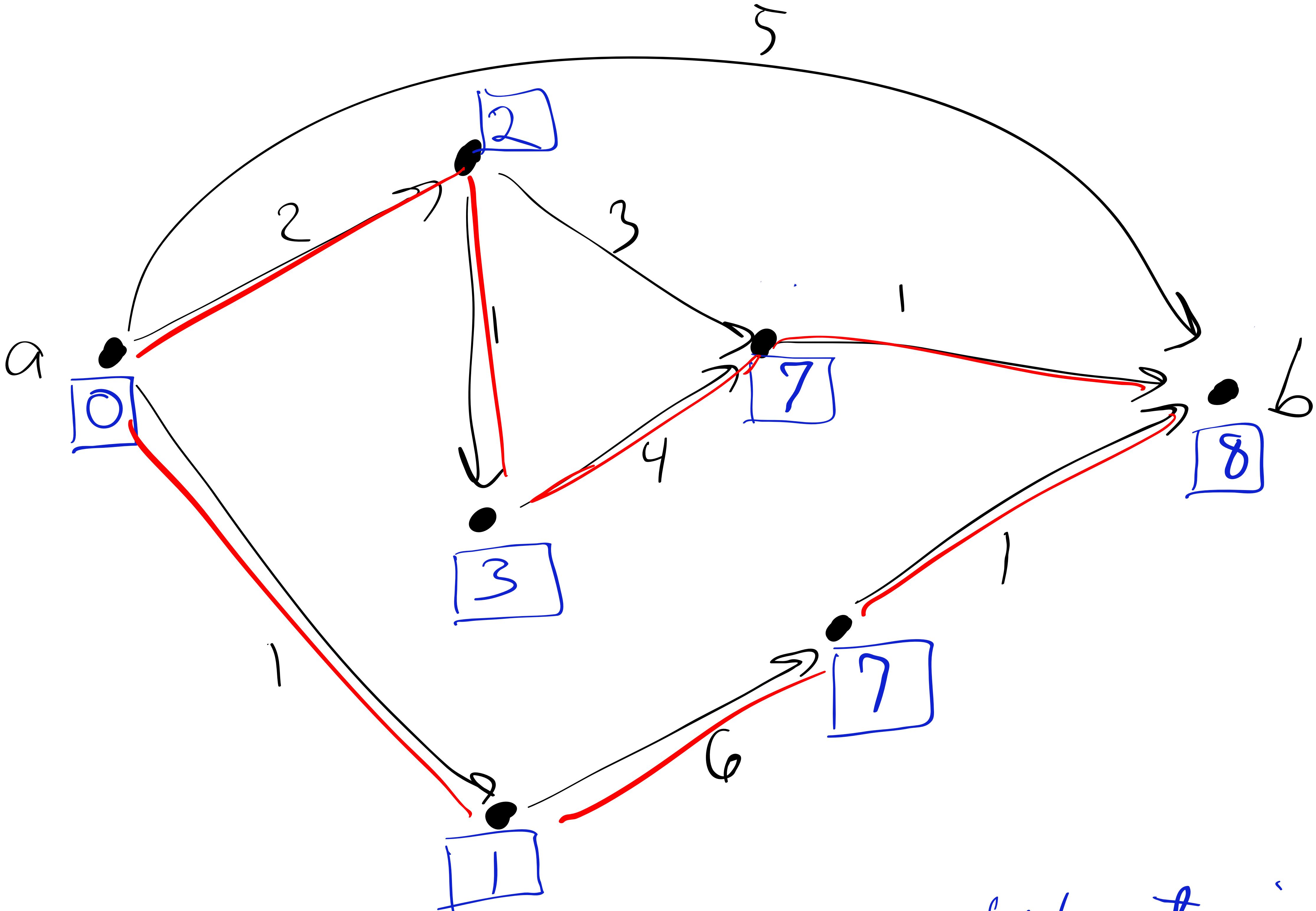
List, or count, all paths from a to v in a digraph.

Q: What digraphs from scheduling look like?

(E.g. Hope not like Petersen graph.)


b
Built a howze.
Lay slate
sheet rock
10
10
inspection
5
1
wiring
plumbing
frame roof
1
20
5
Frame Walls
4
Foundation poured
400
a
cleared the land

no circuits



Worst is 8.