Lecture 20: Euler Trails (trail = no repeated edges)

Let $G$ be a graph. An Euler trail is a trail that covers every edge.

**Theorem:** If $G$ is connected, it has an Euler trail iff it has zero or two of odd degree.
How to find an Euler Trail if there are 2 odd vertices.

Start at an odd vertex, use Fleury's Algorithm (forced to end at the other odd vertex).

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"Smaller" = fewer vertices.

One vertex.

Two vertices.

1. All loops on v.
2. Go to w.
3. All loops at w.
4. ...
pf of better Euler Thm:

\[ G_1 = \text{"G plus } e^{"} \]

Previous Thm says \( \exists \) Euler Circuit for \( G_1 \). Remove \( e \) from the circuit, "get a Euler trail for \( G \)."
prove: Euler Theorem (First version) again. By induction on # of vertices:

\[n = k + 1\]
\[n = k\]
\[(\text{ok for } n = k) \& (\text{ok for } n = 2)\]
\[(\text{ok for } n = k + 1)\]
\[ \Delta = \{v, w\} \]

\[ n = k + 1 \text{ vertices} \]

a) \text{ Find a circuit here.}

b) \text{ If a graph here.}