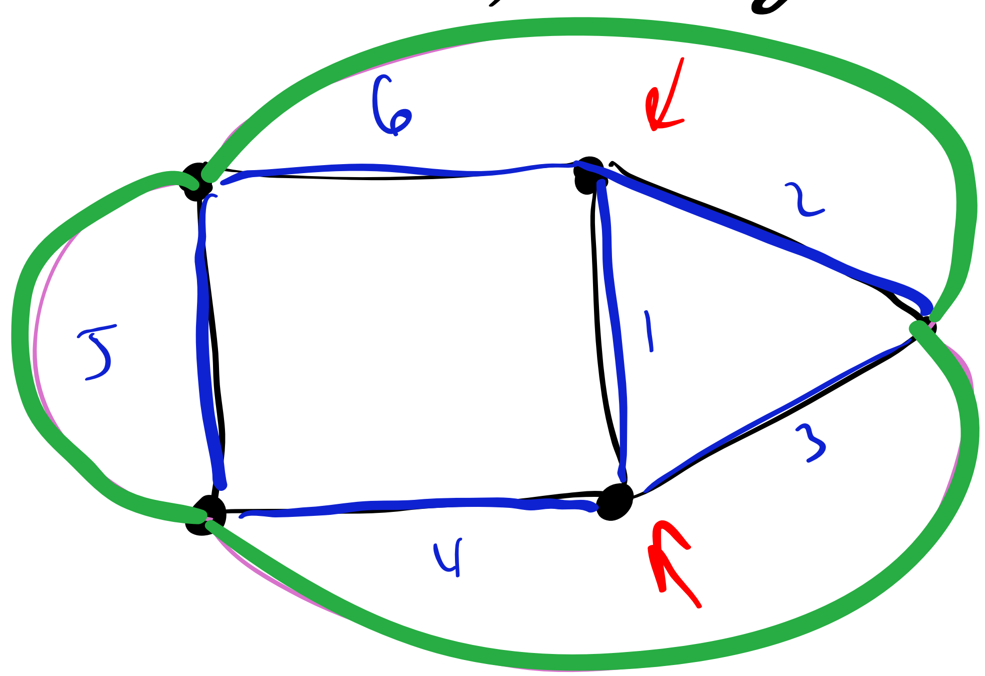
Lecture 20. Euler Trails (trail = no repeated edges)

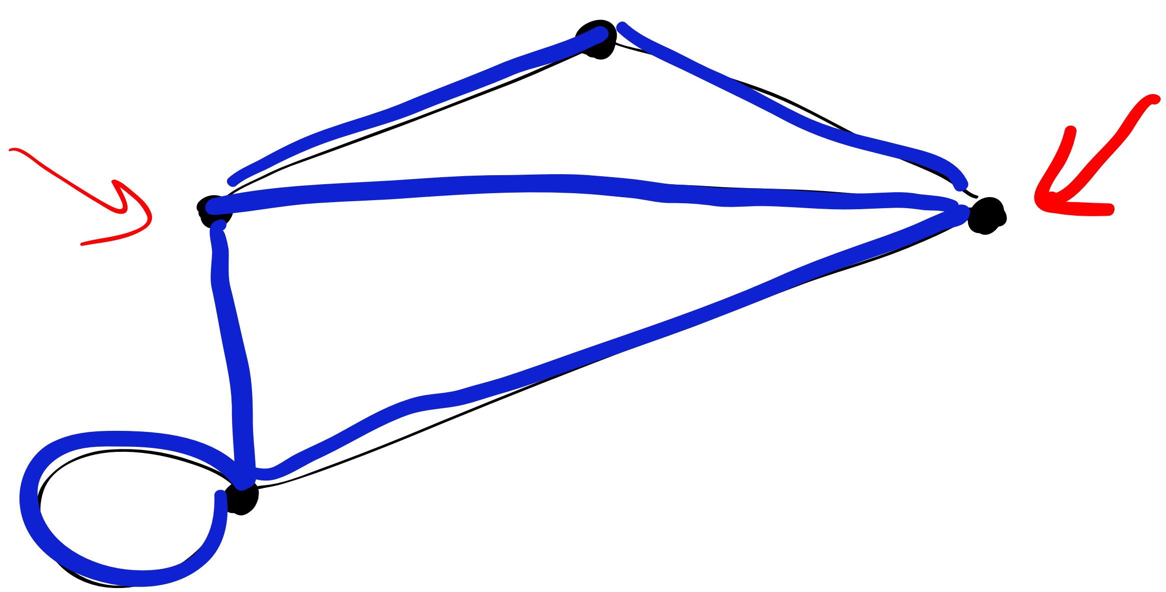
G a graph. An Euler trail is a trail that covers every edge.

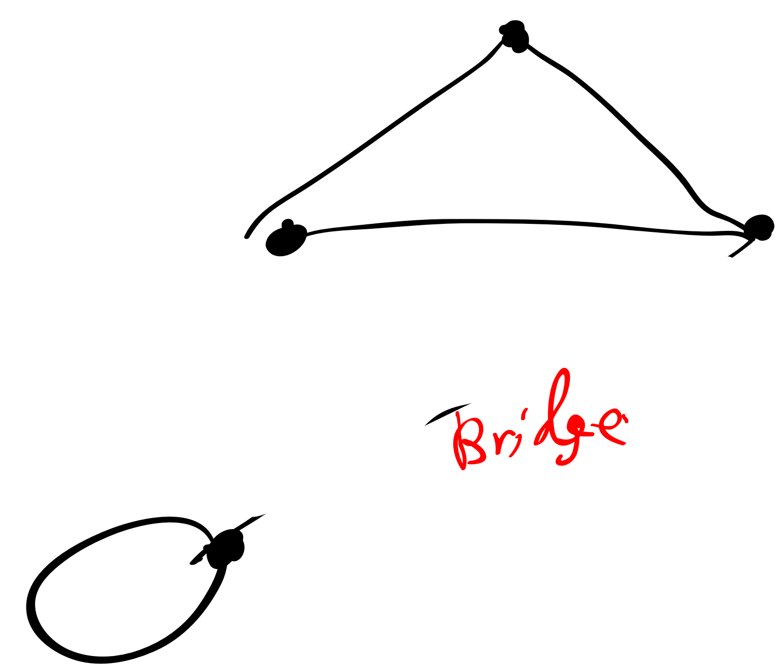


Theorem: If G is connected, it has an Euler trail iff it has zero or two of odd degree.

How to find an Euler Trail if ∃ 2 odd vertices.

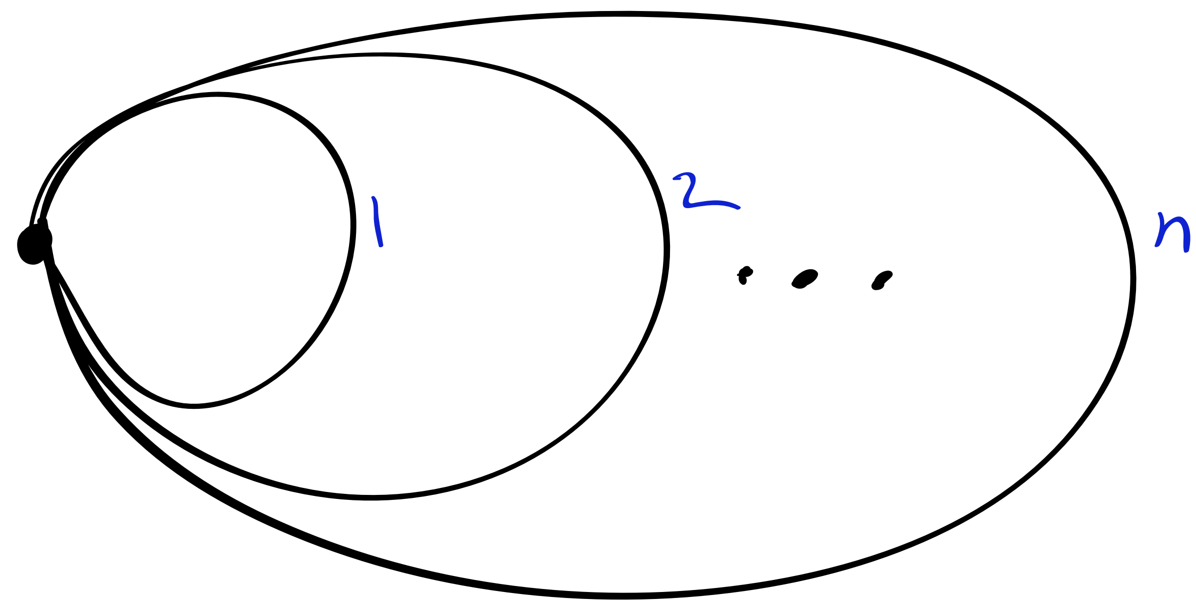
Start at an odd vertex, use Fleury's Algorithm. (Forced to end at the other odd vertex.



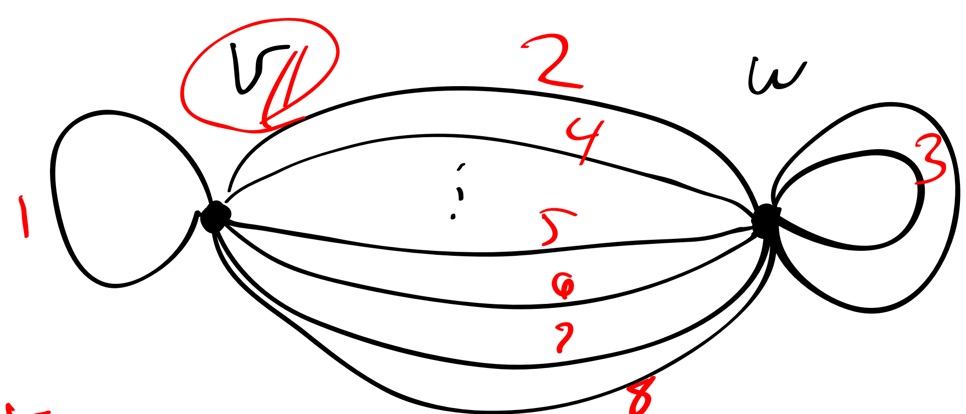


"Smaller" = fewer vertices.

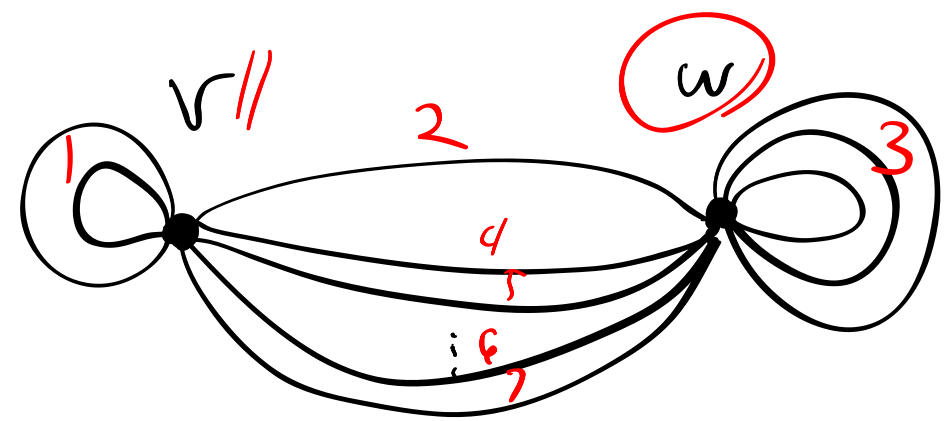
one vertex?



two vertices.



even # of "vw" edges.



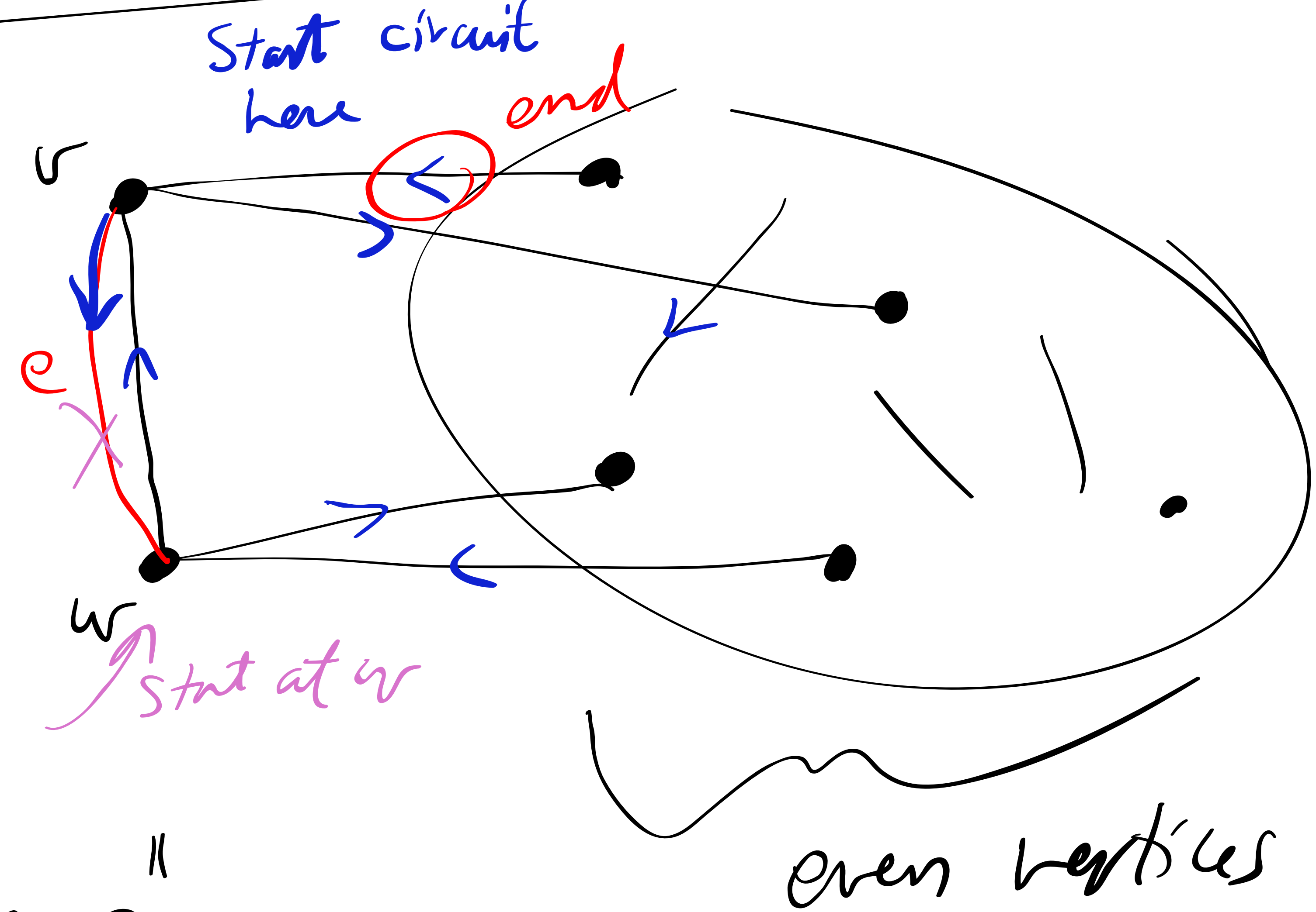
odd # of "vw" edges

① all loops on v.
② Go to w.
③ all loops at w.
④ ←, →, ←.
, ..., ←

pf of better Euler Thm:

G:

(know Thm for 0 odd vertices)

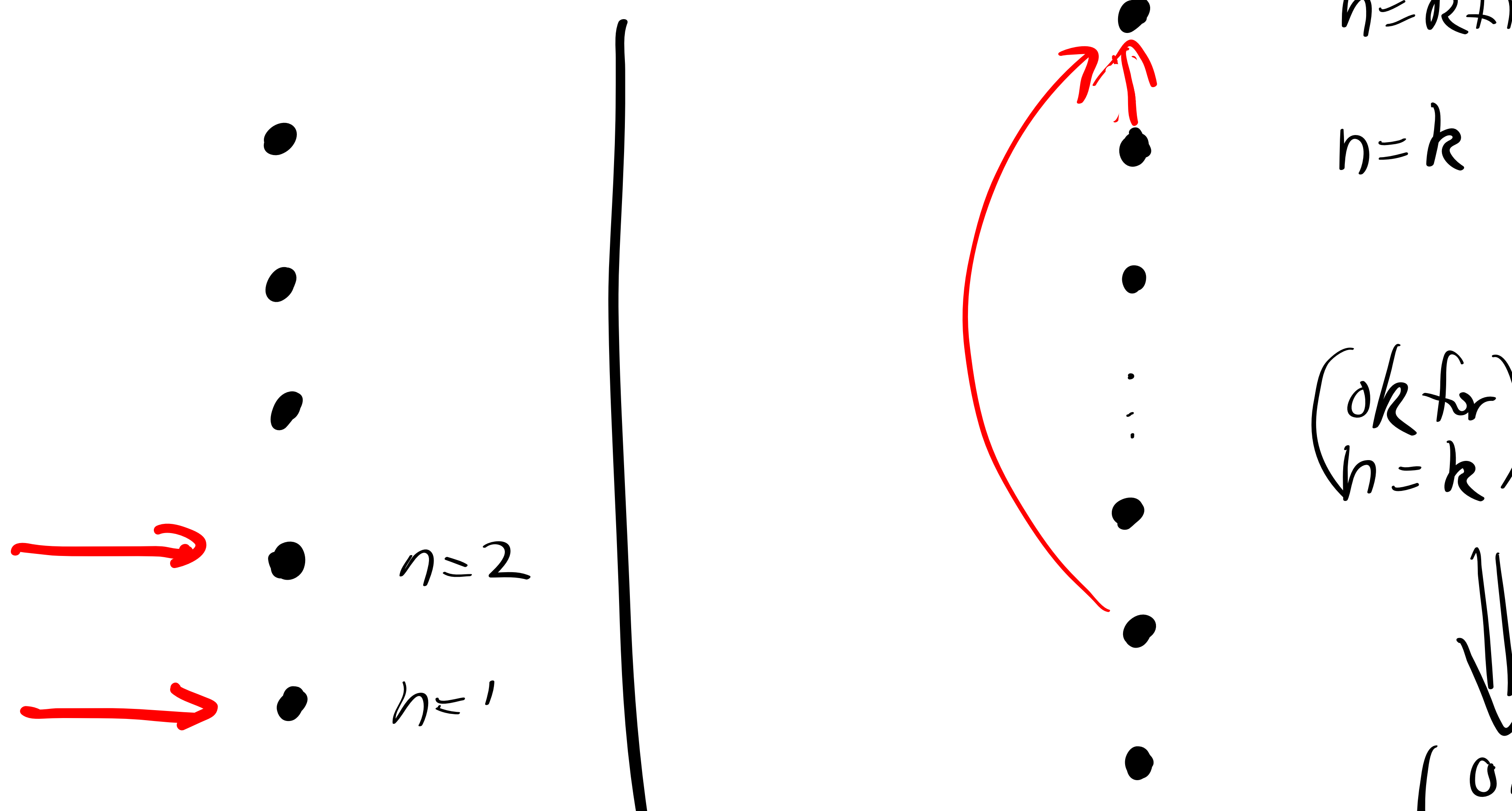


G_1 = "G plus e"

Previous Thm says $\exists$ Euler Circuit for G_1. Remove e from the circuit, "get a Euler trail for G."

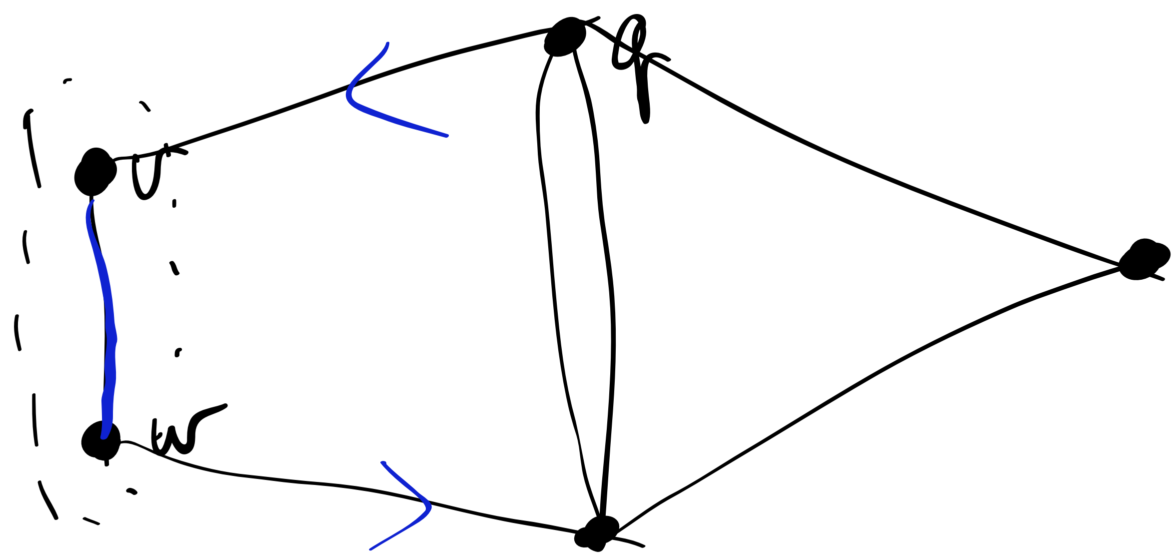
Prove: Euler Theorem (First version) again.

By induction on # of vertices:



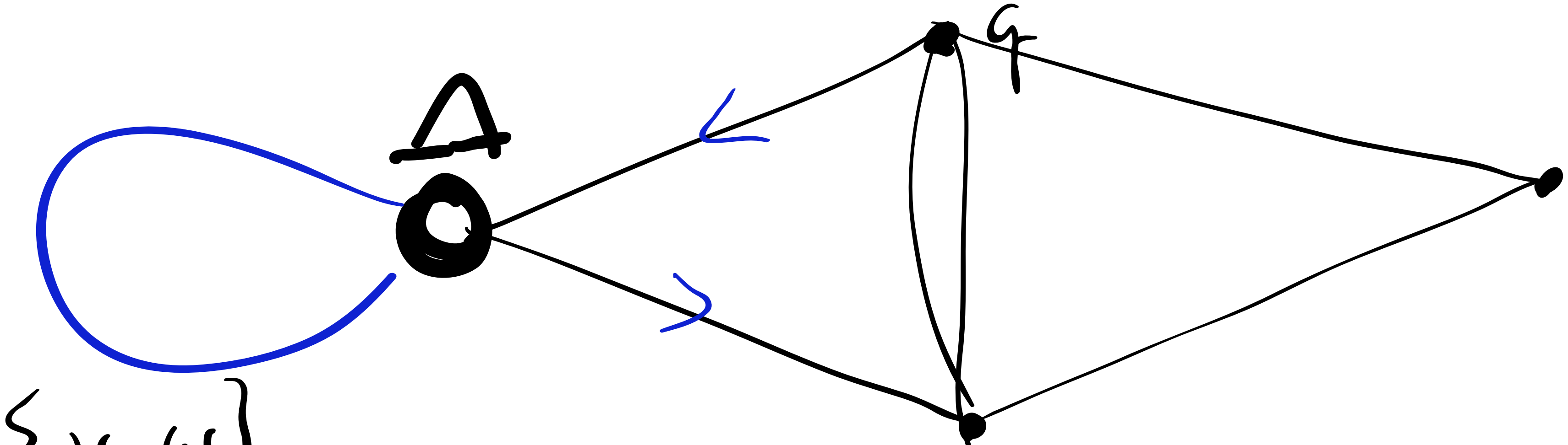
$$\begin{pmatrix}\text{ok for}\\ n=k\end{pmatrix} \,\&\, \begin{pmatrix}\text{ok for}\\ n=2\end{pmatrix} \Longrightarrow \begin{pmatrix}\text{ok for}\\ n=k+1\end{pmatrix}$$

$n = k+1$ vertices



$\therefore$ $\exists$ a circuit here.

Find a circuit here



$\Delta = \{v, w\}$

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