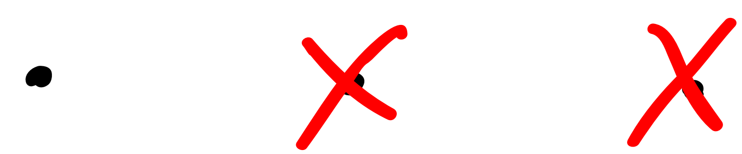
Lecture 19 Euler Circuits.

Thm: If G is connected and has no vertices of odd degree, then G has an Euler Circuit.

pf by induction?

Claim: Thm is true for all such G with n edges.

Base Case: $n=0$.



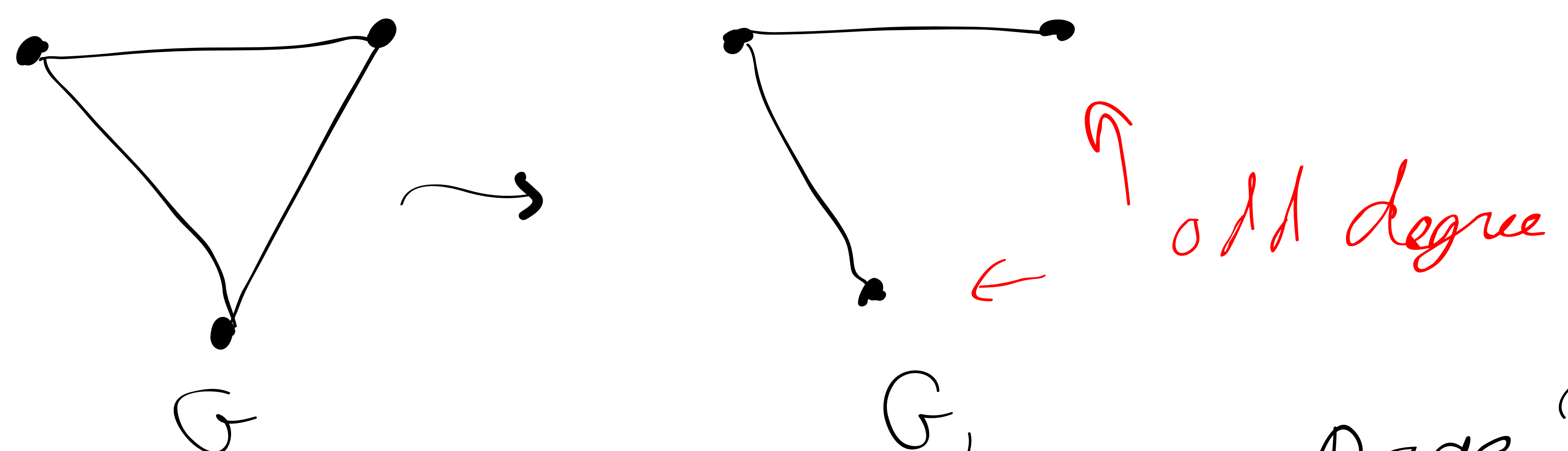
Path = "".

Assume Thm true for $n = 1, \dots, k-1$. Suppose

G has $n = k$ edges, connected, no odd vertices,

$G_1 = G \setminus \{e\}$, for some random edge.

G_1 has $k-1$ edges, so thm applies.



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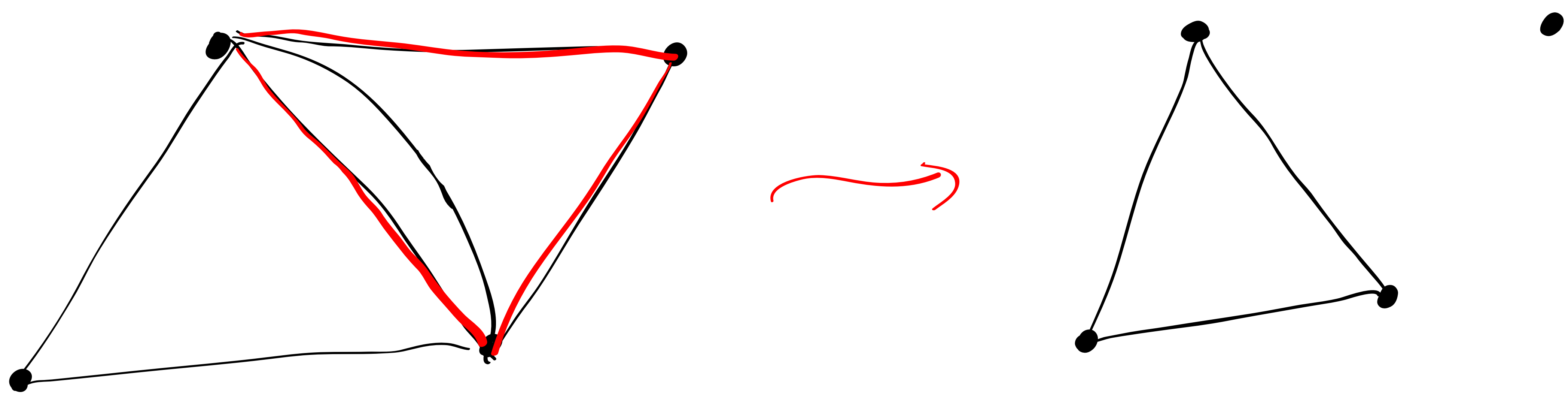
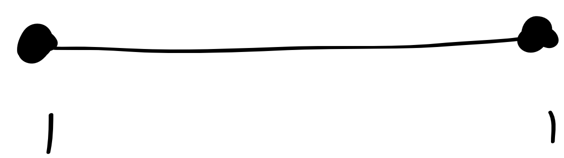
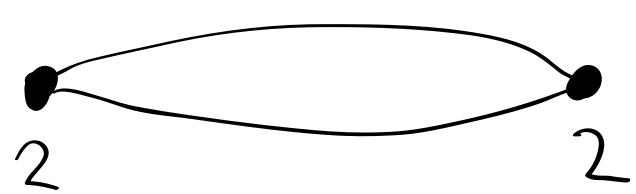
G

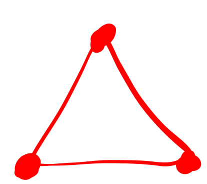


How can I derive a "smaller" graph from G, and keep "connected", "no odd vertices"!

— x —

subtracting "  " failed

Remove "  "

or C_k. 

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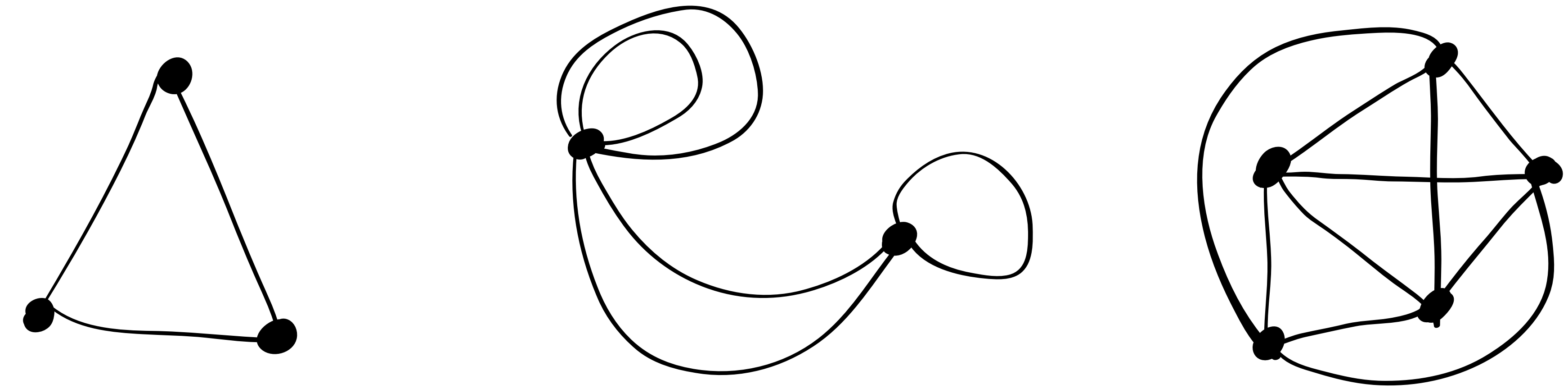
G:



<u>Thm</u>: If G has <u>no odd vertices</u>, then there is a disjoint union of ~~cycles~~ circuits in G that contains each edge once.

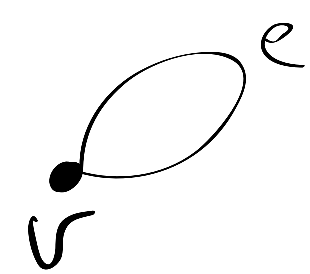
<u>pf</u>: By strong induction on the number of edges.

<u>n=0</u>: 

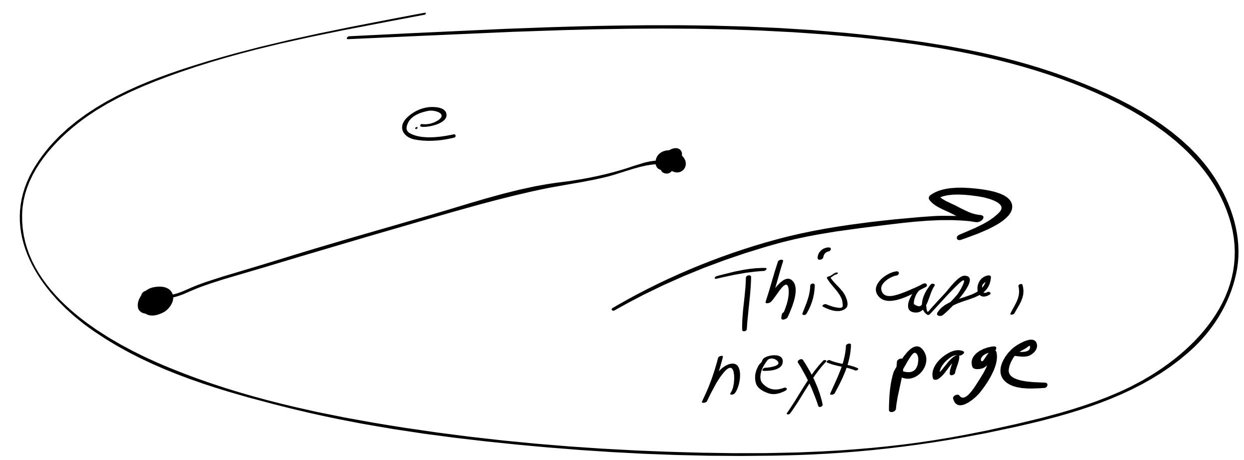
Done with zero cycles.

(*) Assume Thm holds for graphs with $n = 0, 1, \dots, k-1$ edges, $k \geq 1$.

Suppose we're given G, has k edges and no odd vertices. $k \geq 1$, so pick an edge e.

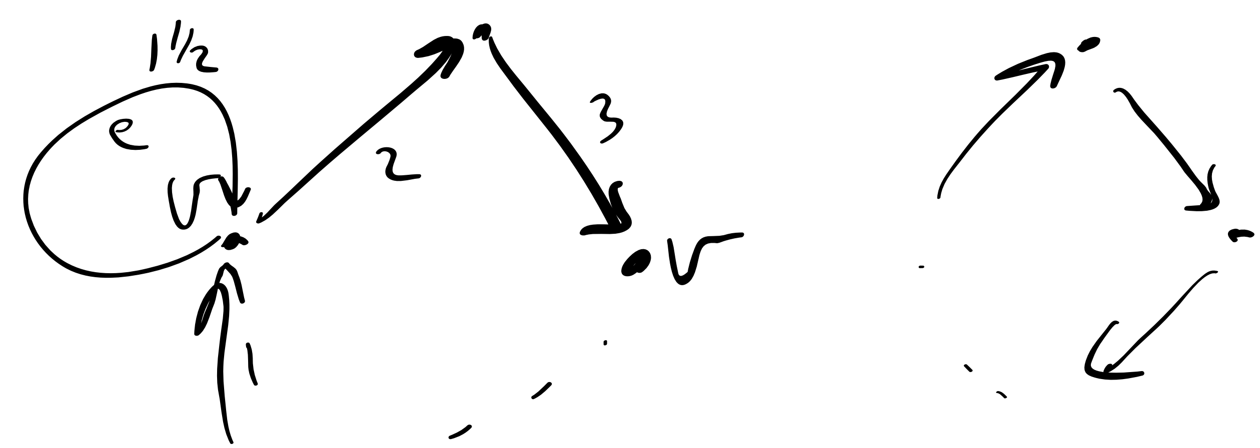
G contains: 

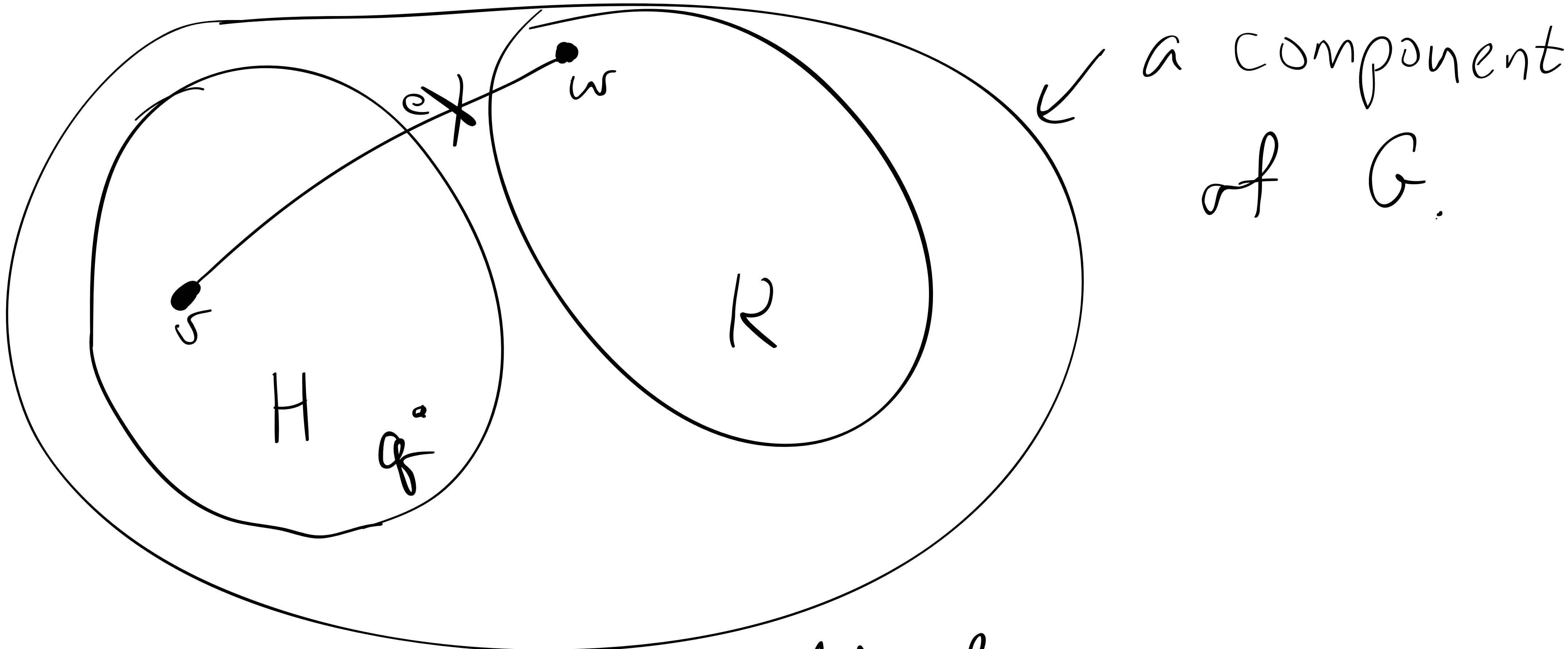
or



loop case $\longrightarrow$ $G - \{e\}$ has same components as G.

By (*), have circuits, disjoint, with union including all edges. "Splice" into a circuit.





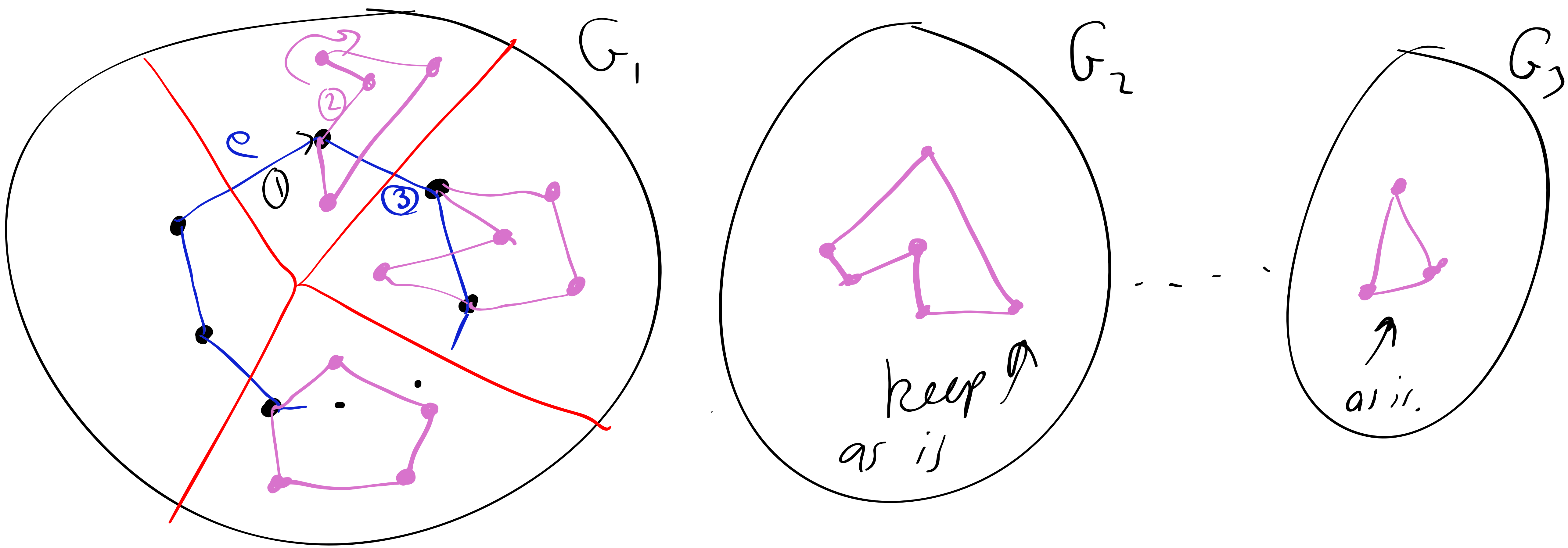
H has v, odd degree.

# of odd vertices is even, always,

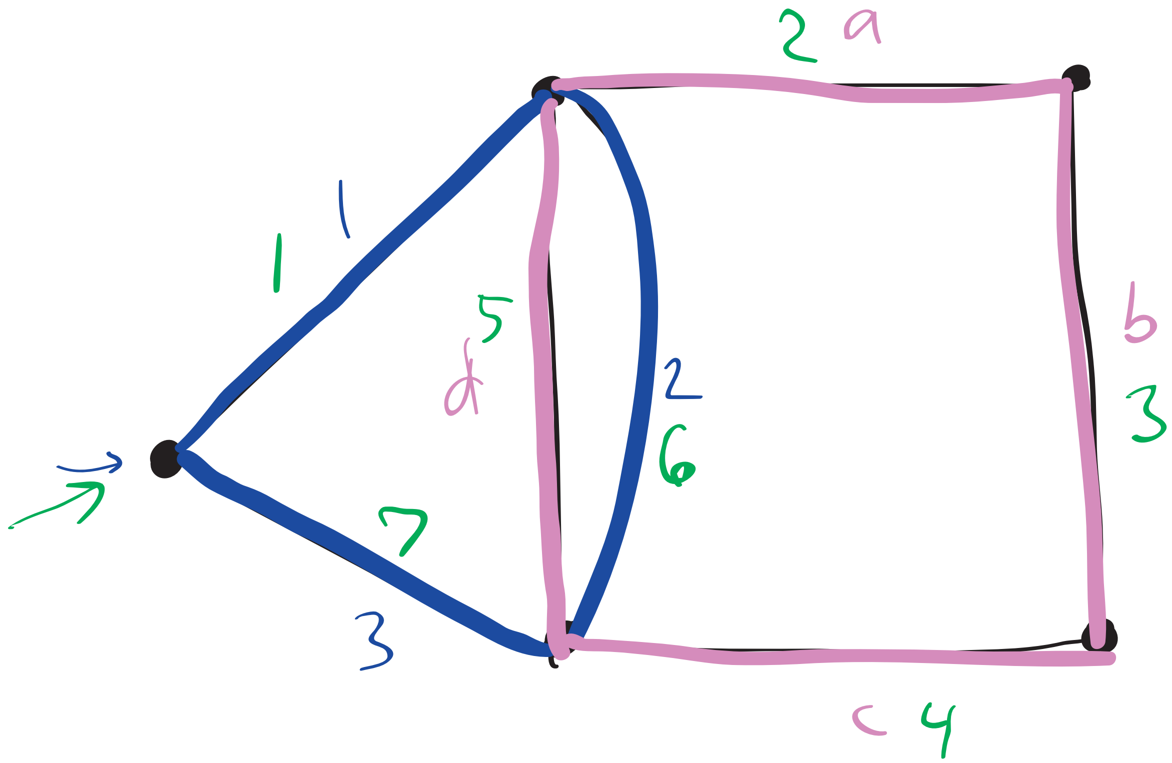
So ∃ vertex of odd degree.

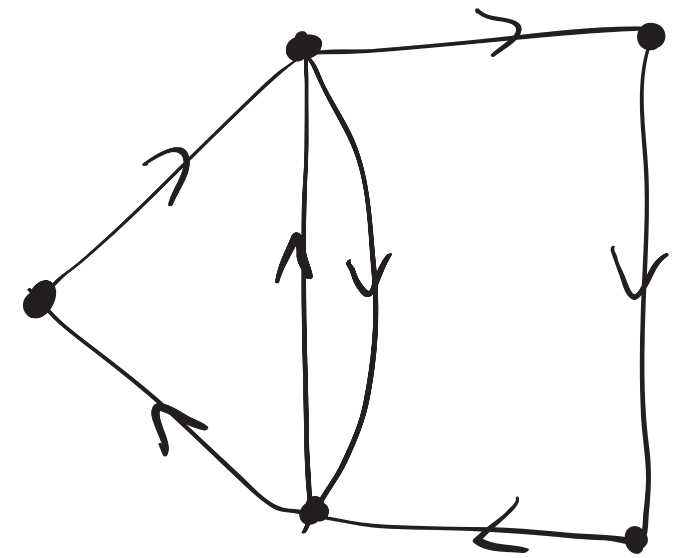
∴ G has vertex of odd degree. $\Rightarrow\Leftarrow$.

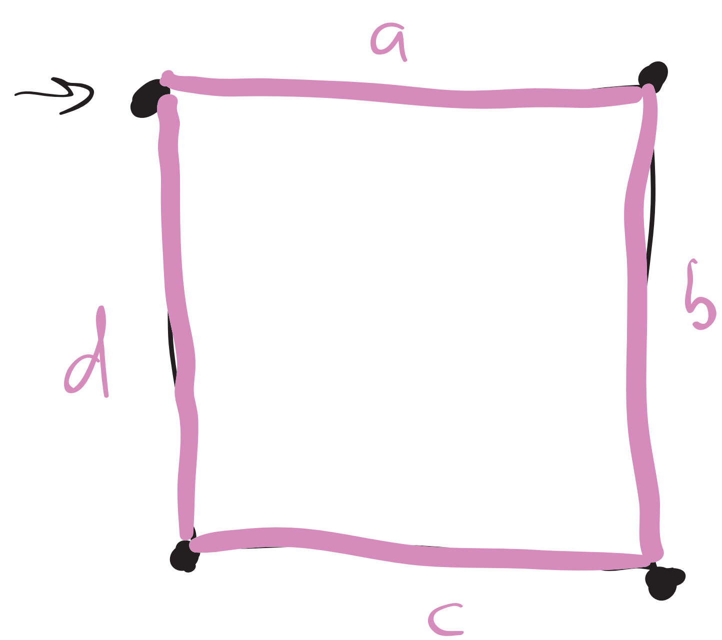
Can't have a bridge. So e is in a cycle.



Now Remove the cycle, call the remainder $\tilde{G}$. The purple circuits exist since $\tilde{G}$ has fewer than k edges. Alternate blue and purple to finish off the component containing e.







null circuit.