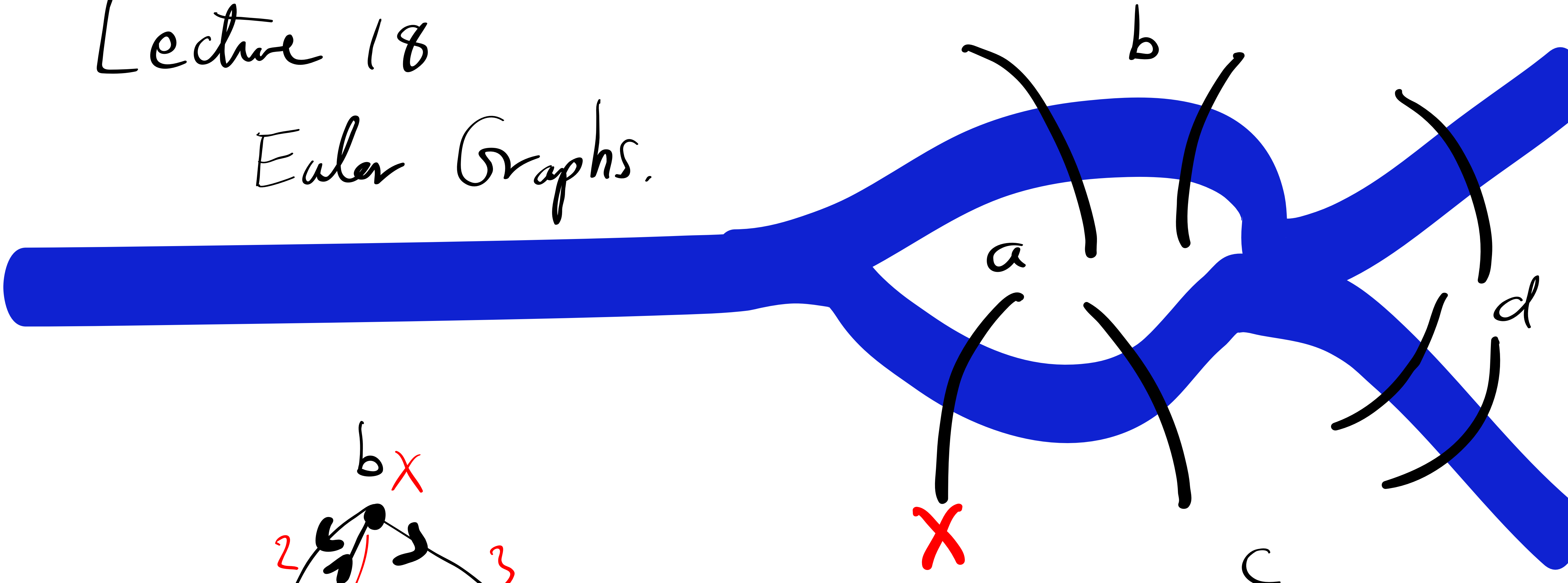
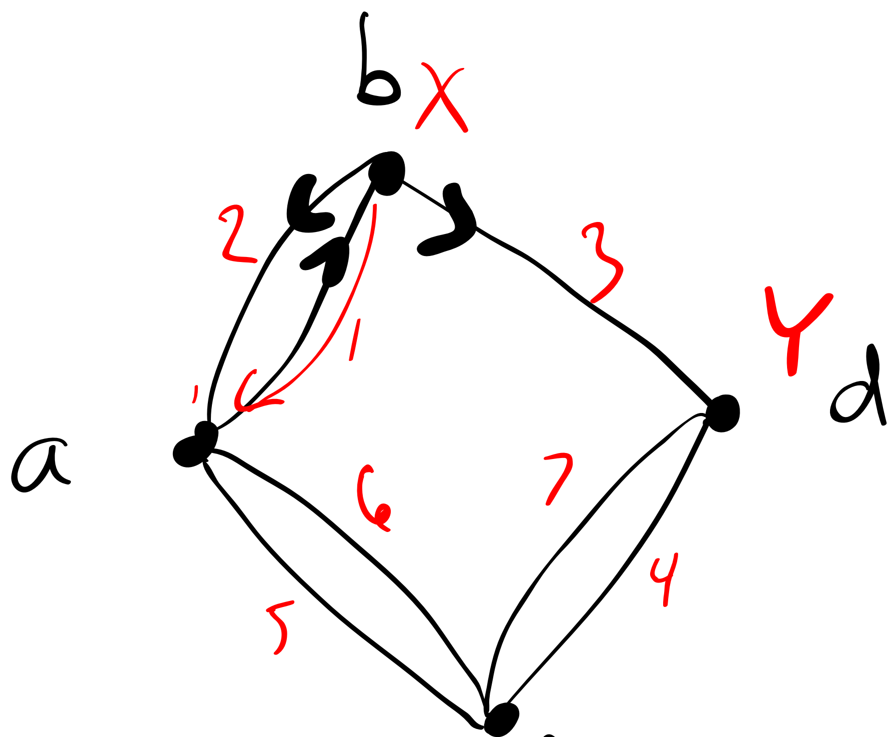
Lecture 18

Euler Graphs.





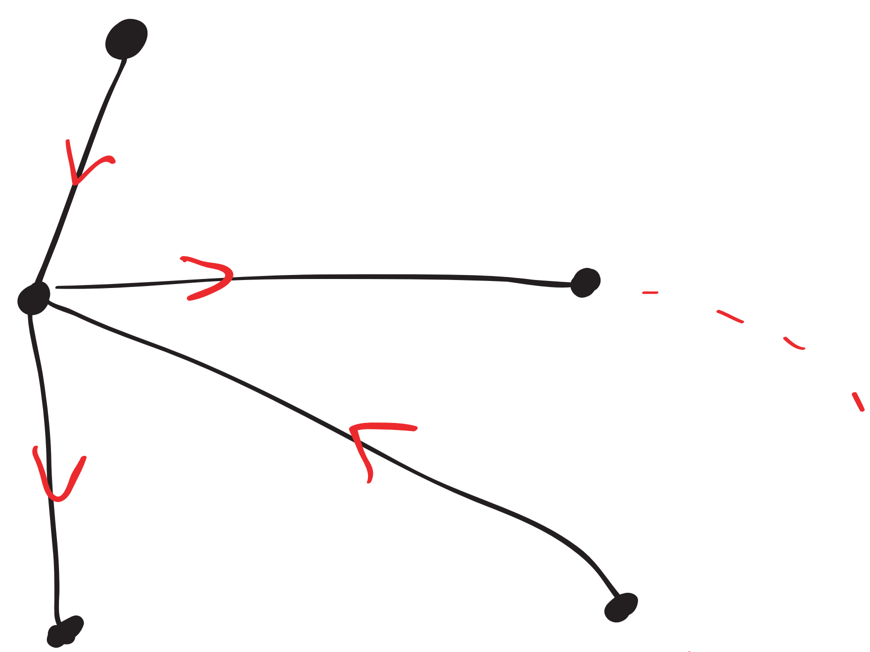
Is there a cycle that contains every edge, no repeats?

deg(b) = deg(d) are odd.

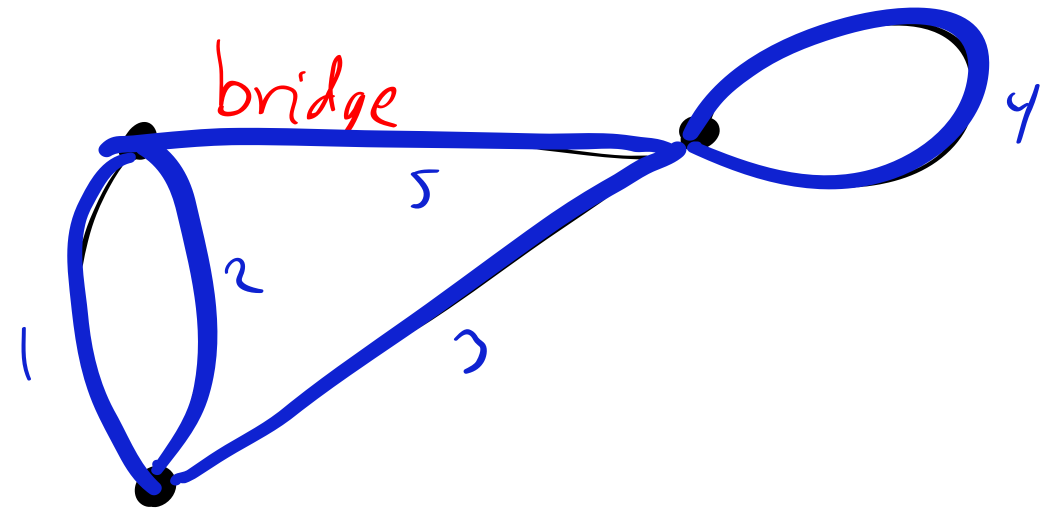
Theorem: If G is connected and has no vertices of odd-degree, then G has an Euler circuit.

3 proofs: 2 by induction, on size of graph

1 "check an algorithm" → by induction on steps in a loop.



Chose a bridge only if forced ("bridge" in remaing graph)


bridge
5
2
1
3
4

Induction. $1+2+\cdots+n=\dfrac{n(n+1)}{2}$ (*)

show (*) ok for the smallest n:

$$1 \overset{?}{=} \frac{1(1+1)}{2} = \frac{1\cdot \cancel{2}}{\cancel{2}} = 1 \checkmark.$$

Assume (*) is ok for a certain size of n:

Assume $1+2+\cdots+k=\dfrac{k(k+1)}{2}$

Add $k+1$ both sides:

LHS: $1+2+\cdots+(k+1)$. RHS is: $\dfrac{k(k+1)}{2}+k+1$

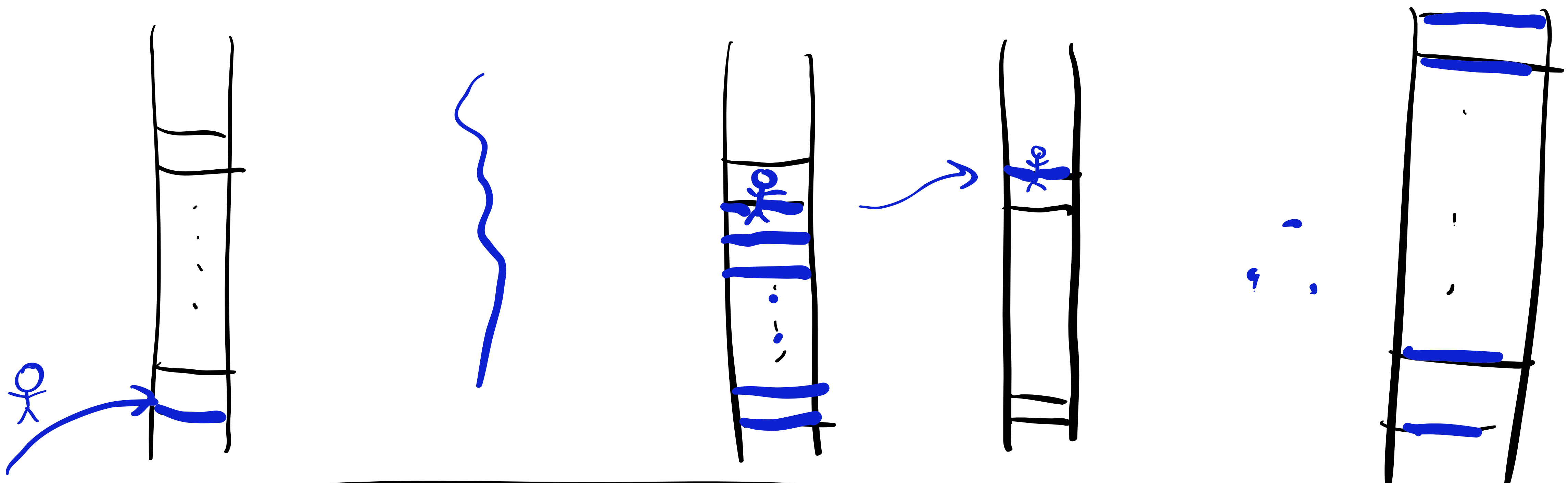
$$= \frac{k^2+k+2k+2}{2} = \frac{(k+1)(k+2)}{2}$$

if $n=k+1$ $\quad = \dfrac{n(n+1)}{2}$

$\therefore$ (*) ok for next size of n.

Induction in graph theory

Usually: Use strong induction.



Prove every # factors into primes:

Suppose $k = p_1 \cdot p_2 \cdot \cdots \cdot p_n$.

$- (k+1) = ??$

Factor 12: useful to know $6 = 2 \cdot 3$.

$\underline{\times 2}$ $12 = 2 \cdot 2 \cdot 3$

$p = 2$: is prime.

Assume every number between 2 and k can be factored.

if $k+1$ is prime - We're done.

o/w: $k+1 = a \cdot b$, with $2 \le a \le k$, $2 \le b \le k$.

$\therefore$ $a = p_1 \cdots p_n$

(x) $b = p_{n+1} \cdots + p_m$

$k+1 = a \cdot b = p_1 \cdots p_n p_{n+1} \cdots p_m$.

✓

edges,
or
vertices,
#(edges + vertices).