# Lecture 16. Change: ch 5 — only § 5.2

— Finite state Automata.

## Last class - Audio lost at end.

Theorem (from last time). If $G$ is a connected graph and $G$ has no bridges, then $G$ is orientable.

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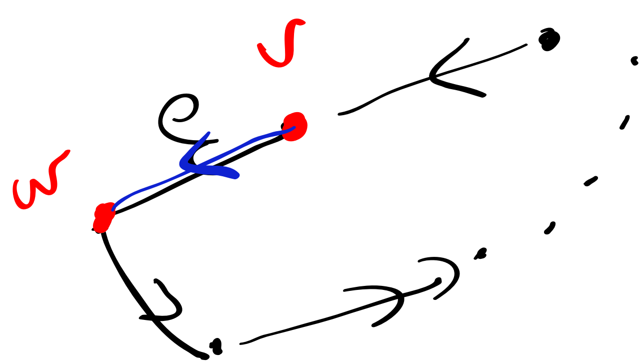
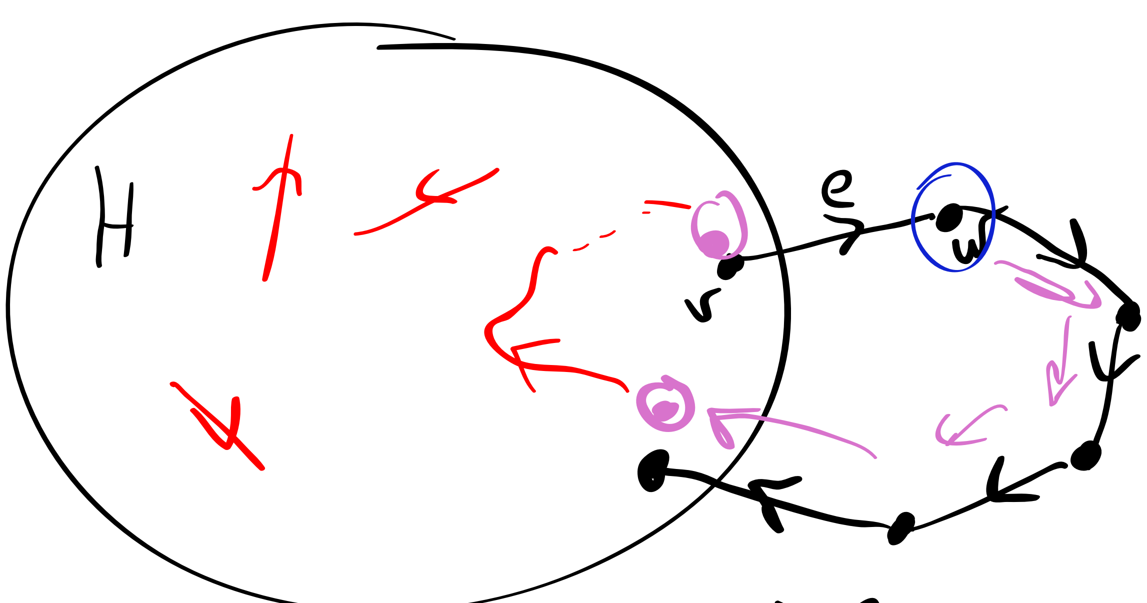


pf: Assume $G$ is connected, has no bridges, yet $G$ is not orientable. Pick a subgraph $H$ that $H$ has a many vertices as possible with $H$ being orientable. Since a subgraph with 1 vertex is orientable, $H$ is nonempty, and $H \neq G$.

Pick an edge $e$ from $v$ to $w$ with $v$ in $H$ and $w \in H$.

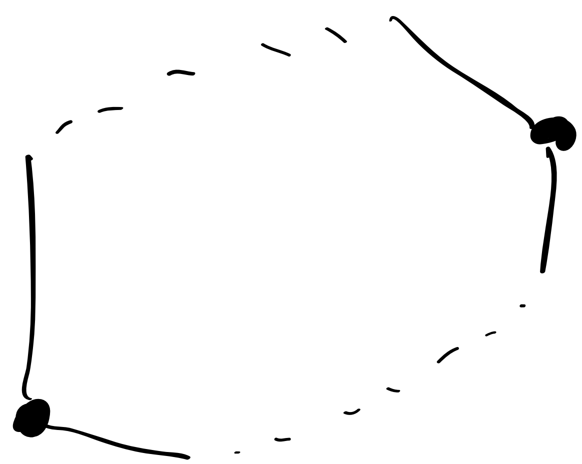



There is a path from $w$ to $v$ in $G$, avoiding $e$.

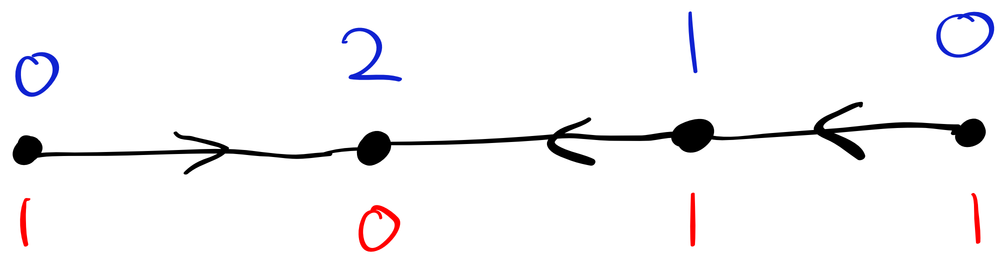
$\tilde{H}$ will be $H$ plus $e$ plus the other path from $w$ to $v$, but stopping when we hit $H$.

$\tilde{H}$ is orientable, and bigger than $H$.

Connected, no bridges:



in-degree & out degree sequence.




- in-deg. seq. 0, 0, 1, 2
- out-deg. seq. 0, 1, 1, 1

is there a $v$ with $in(v) = 2$, $out(v) = 1$ ?

no!

(0,1), (0,1), (0,2), (1,1)

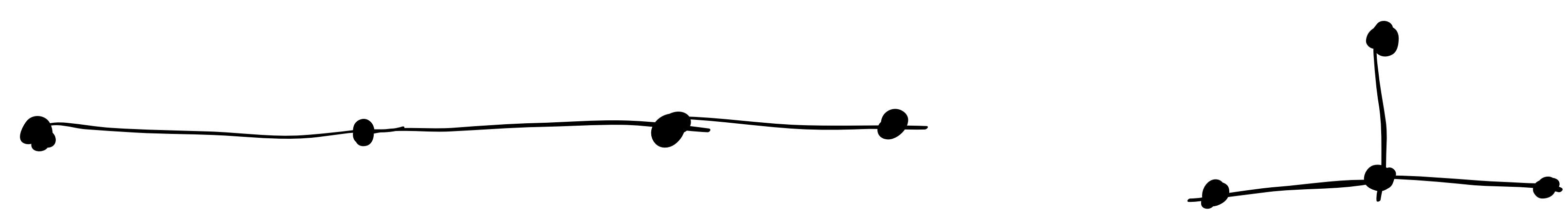
Q: How many non-isomorphic simple, connected digraphs are there with 3 arcs, 4 vertices.

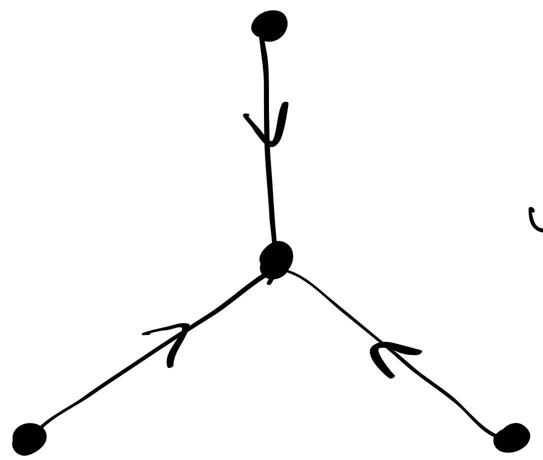
if D is such a digraph, what about the underlieing graph? connected ✓

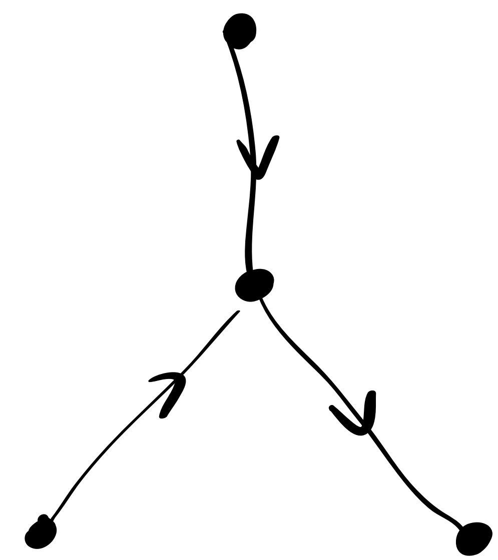
no loops.

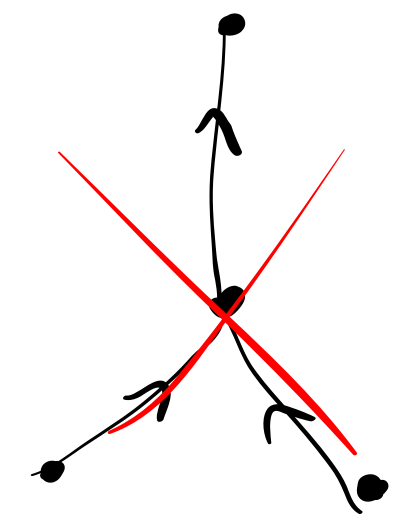
need 4 edges to get connected.

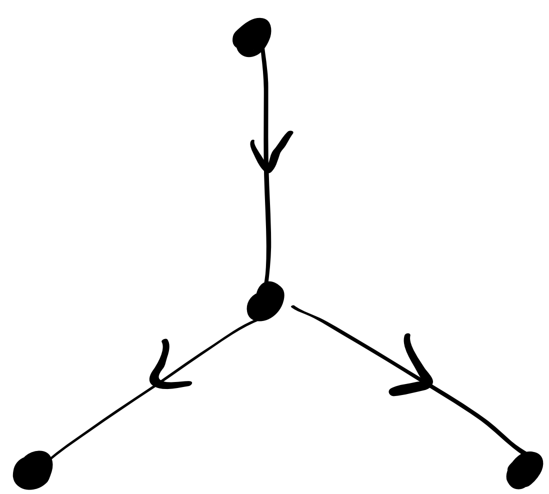
Simple: ✓

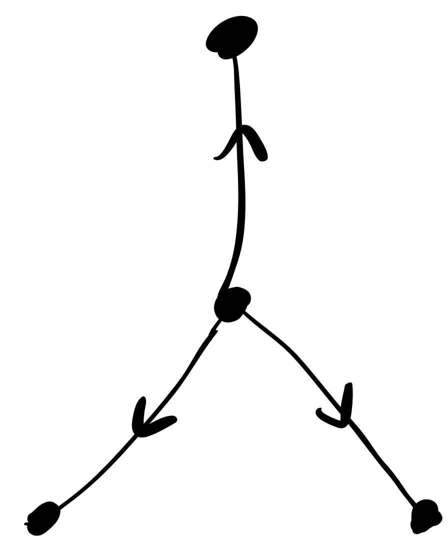


 ✓



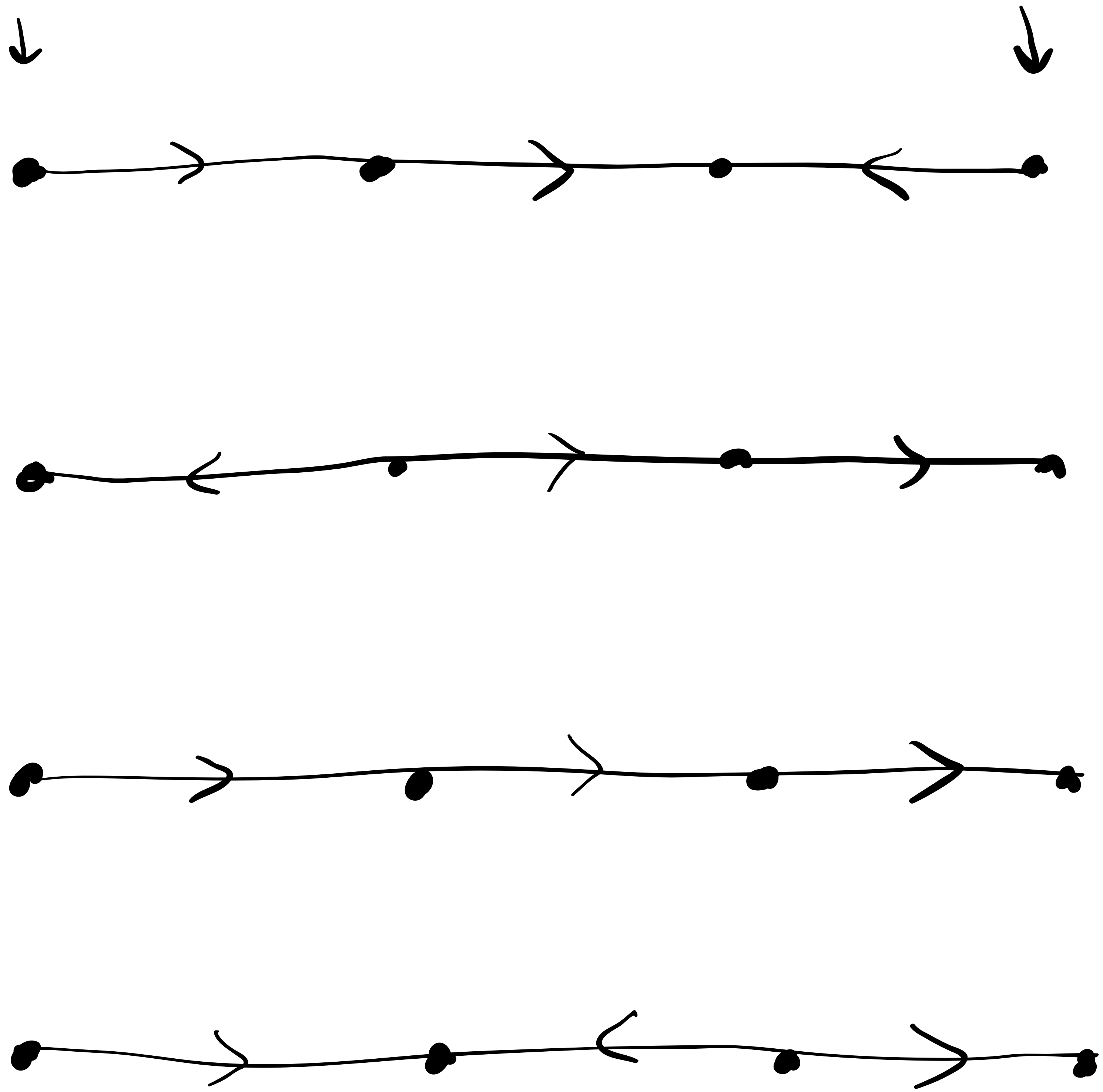




 ✓

$\deg(v) = 3 \longleftrightarrow$

| | | | | |
|---|---|---|---|---|
| in | 3 | 2 | 1 | 0 |
| out | 0 | 1 | 2 | 3 |



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