Lecture #15. Digraphs

$$v \bullet \xrightarrow{\quad e \quad} \bullet\, w$$

v is adjacent to w.

e is incident from v

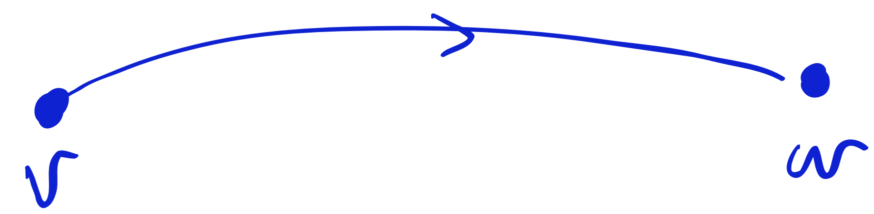
e is incident to w.

<u>Def</u>: in-degree of v as the number of arcs incident to v.

out-degree of v as the number of arcs incident from v.

$$\sum_{v \text{ in } V(D)} \text{in}(v) = \sum_{v \text{ in } V(D)} \text{out}(v) = \#\text{ of edges in } D.$$

(v, w)

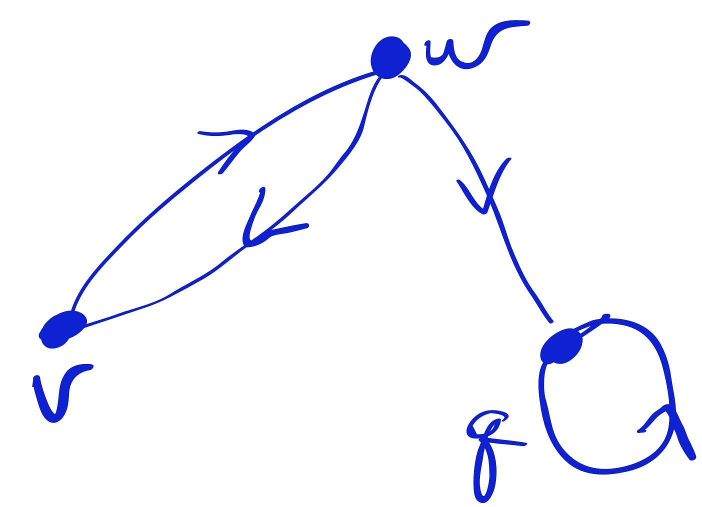


This arc is from v to w.

D consists of: a set $V(D)$, and a(n) (unordered) list of ordered pairs $A(D)$.

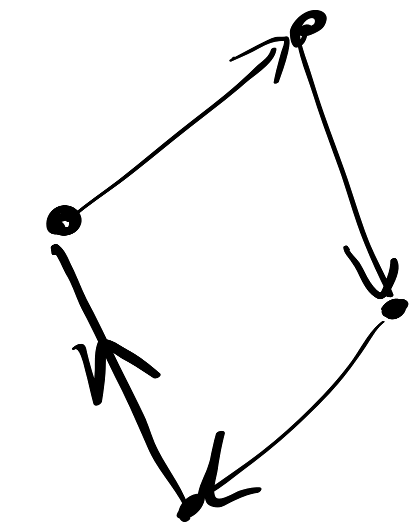
$V(D)$: $\{v, w, q\}$

$A(D)$: (v,w), (w,v), (w,q), (q,q)

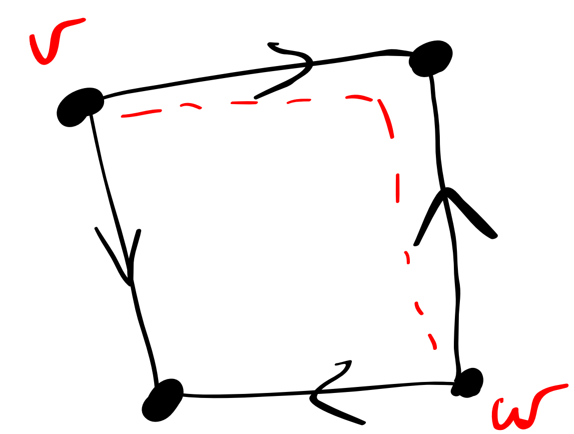


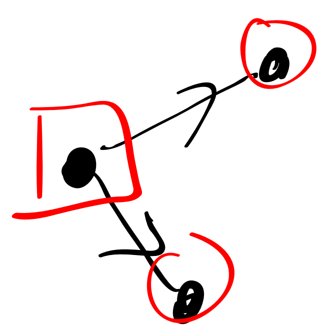
Def Walk $\supseteq$ trail $\supseteq$ path

closed walk $\supseteq$ closed trail $\supseteq$ cycle



Def: D is **strongly connected** if, given v, w in $V(D)$, there is a path from v to w.





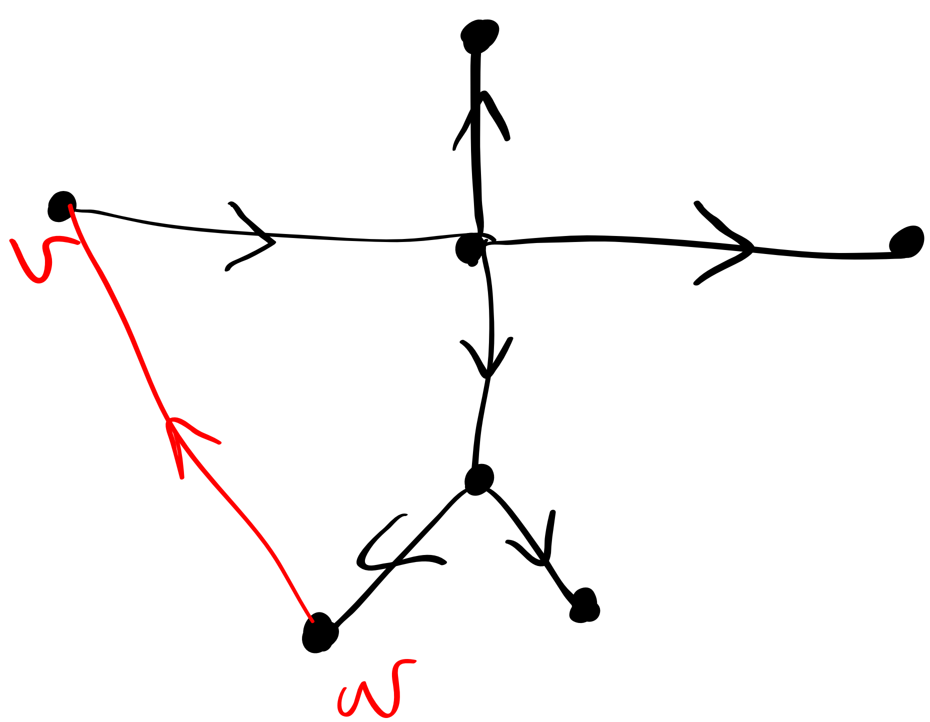
Ch 4

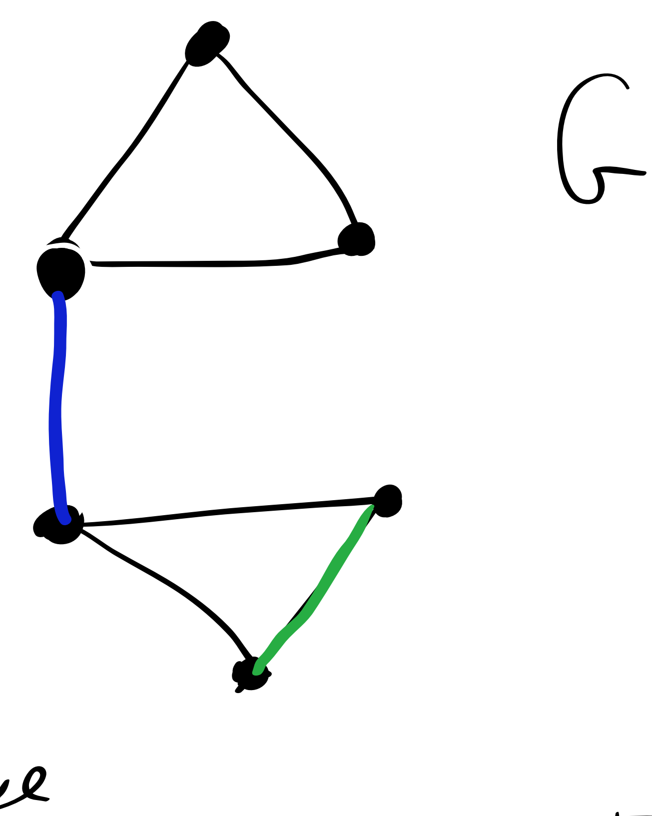
\# 4.5, 4.8, 4.24 (a), 4.24 (b)

"try contradiction"

See lecture notes where: v and w are connected by a walk, then they are connected by a path.

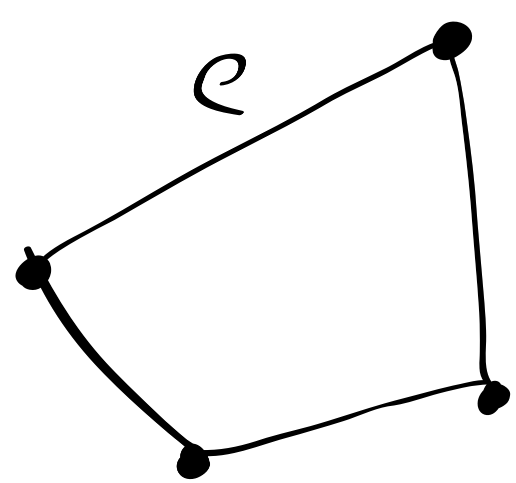
Def: A graph G is orientable if there is a digraph D with G the underlying graph of D, and D is strongly connected.





Def: An edge e is a graph is a **bridge** if $G \setminus \{e\}$ is disconnected, but G is connected.

Lemma. If G is connected and has no brides, then every edge is in a cycle.



Theorem. Suppose G is connected. Then G is orientable $\iff$ G has no bridges.

[Examples, before a proof.