REVIEW #1

**Problem 1.** Find all partitions of \{0, 1, 2, 3\} into two sets, one of which is a singleton. Write out the associated equivalence relations as tables.

**Problem 2.** Find an example of a function

\[ f : \{1, 2, 3, 4, 5\} \rightarrow \{1, 2, 3, 4, 5\} \]

such that \( f \circ f \circ f = \text{id} \) but \( f \circ f \circ f \) is not the identity function.

**Problem 3.** Let

\[ A = \{(0, 0), (1, 2), (0, 2), (1, 1), (2, 2)\} \]

and define

\[(a, b) \preceq (c, d)\]

to mean

\[ a \leq c \text{ and } b \leq d. \]

Draw the Hasse diagram associated to the partial order \( \preceq \).

**Problem 4.** Suppose \( \sim \) is the equivalence relation indicated by this table:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) What is \( \overline{3} \)?
(b) What is \( \overline{10} \)?
(c) Is \( 11 \in \overline{T} \)?

**Problem 5.** Let \( f \) be the invertible function indicated by this table.
(a) What is \( f \circ f \circ f(2)? \)
(b) What is \( f \circ f^{-1} \circ f(2)? \)
(c) Is \( f \) one-to-one?

Problem 6. It is not true that for all sets \( A, B \) and \( C \) we have the equation
\[
A \cap (B \setminus C) = B \setminus (A \cap C).
\]

(a) Find an example where this equation is true.
(b) Find an example where this equation is false.

Problem 7. Draw a Hasse diagram such that for the associated partial order there are two elements \( a \) and \( b \) that have at least one common upper bound but that have no least upper bound.

Problem 8. Is \( R \) transitive? Say why, of course.