HOMEWORK #4

Problem 1. Using Euclidean algorithm, find the GCD of each pair:

(a)
$$13, 34$$

(b)
$$1559, 31$$

(c)
$$12008, 12$$

(d)
$$\left(\left(2^4 3^3 5^{10}\right) + 8\right), 12$$

Problem 2. Find two different ordered triples of natural numbers
$$(k, m, n) \neq (r, s, t)$$
so that
$$24^k 54^m 36^n = 24^r 54^s 36^t.$$

Problem 3. Here is how the Eucliean algorithm runs to calculate
$$\text{GCD}(7581, 7506) = 3.$$
Since
$$7581 = 1 \times 7506 + 75$$
$$7506 = 100 \times 75 + 6$$
$$75 = 12 \times 6 + 3$$
$$6 = 2 \times 3 + 0$$
we have
$$\begin{aligned} \text{GCD}(7581, 7506) &= \text{GCD}(7506, 75) \\ &= \text{GCD}(75, 6) \\ &= \text{GCD}(6, 3) \\ &= 3 \end{aligned}$$
Using these calculations, find pairs of integers to solve each of the following, in turn:

(a)
$$u75 + v6 = 3.$$

(b)
$$u7506 + v75 = 3.$$

(c)
$$u7581 + v7506 = 3.$$

Problem 4. Number 4.2.10(b,d)

(a) Notice

$$2|60 \text{ and } 12|60$$

and yet

$$24 \not| 60,$$

so this is a no.

(b) Yes. If b divices a and c divides d,then for some whole numbers q and q_1 we have $b = qa$ and $d = q_1 c$. Therefore

$$bd = (qq_1)ac$$

and so ac divides bd.

Problem 5. Number 4.2.22

Problem 6. Number 4.3.32(c)

Problem 7.

(a) Find all integer solutions to

$$2x \equiv 1 \pmod 9$$

(b) Find all integer solutions to

$$2x \equiv 5 \pmod 9$$

(c) Find all integer solutions to

$$3x \equiv 0 \pmod 9$$