HOMEWORK #4

Problem 1. Using Euclidean algorithm, find the GCD of each pair:
   (a) 13, 34
   (b) 1559, 31
   (c) 12008, 12
   (d) \(((2^43^35^{10}) + 8), 12\)

Problem 2. Find two different ordered triples of natural numbers 
   \((k, m, n) \neq (r, s, t)\)
   so that 
   \(24^k54^m36^n = 24^r54^s36^t.\)

Problem 3. Here is how the Euclidean algorithm runs to calculate 
   \(\text{GCD}(7581, 7506) = 3.\)
   Since 
   \[\begin{align*}
   7581 &= 1 \times 7506 + 75 \\
   7506 &= 100 \times 75 + 6 \\
   75 &= 12 \times 6 + 3 \\
   6 &= 2 \times 3 + 0
   \end{align*}\]
   we have 
   \[\begin{align*}
   \text{GCD}(7581, 7506) &= \text{GCD}(7506, 75) \\
   &= \text{GCD}(75, 6) \\
   &= \text{GCD}(6, 3) \\
   &= 3
   \end{align*}\]
   Using these calculations, find pairs of integers to solve each of the following, in turn:
   (a) \(u75 + v6 = 3.\)
   (b) \(u7506 + v75 = 3.\)
   (c) \(u7581 + v7506 = 3.\)

Problem 4. Number 4.2.10(b,d)
(a) Notice 2 | 60 and 12 | 60
and yet 24 \! | 60,
so this is a no.
(b) Yes. If b divides a and c divides d, then for some whole numbers q and q_1
we have b = qa and d = q_1 c. Therefore
\[ bd = (qq_1)ac \]
and so ac divides bd.

**Problem 5.** Number 4.2.22

**Problem 6.** Number 4.3.32(c)

**Problem 7.**
(a) Find all integer solutions to
\[ 2x \equiv 1 \pmod{9} \]
(b) Find all integer solutions to
\[ 2x \equiv 5 \pmod{9} \]
(c) Find all integer solutions to
\[ 3x \equiv 0 \pmod{9} \]