REVIEW II, MATH 327

Problem 1. Find the GCD of the following pairs:
(a) 12135, 1213505
(b) 144^3, 144^{35}
(c) 144^3, $144^4 + 6$

Problem 2. Find all integer solutions to

$$\begin{aligned} x &\equiv 0 \pmod{13} \\ x &\equiv 4 \pmod{23} \\ x &\equiv 0 \pmod{103} \end{aligned}$$

Problem 3. Find x and y between 0 and 1608 so that

$$\begin{aligned} 12x + y &\equiv 11 \pmod{1609} \\ x - y &\equiv 15 \pmod{1609} \end{aligned}$$

(Note: 1909 is prime.)

Problem 4. Let a_n and b_n be sequences defined by

$$a_1 = 4, \quad a_2 = 1,$$
$$a_n = a_{n-1} + 2a_{n-2}$$
$$b_1 = 2, \quad b_2 = 3,$$
$$b_n = b_{n-1} + 2b_{n-2}$$

For which n is $a_n \geq b_n$?

Problem 5.

(a) How many integers between $10,000$ and $99,999$ (inclusive) are even and do not contain the digit 5?
(b) How many integers between $10,000$ and $99,999$ (inclusive) are odd and do not contain the digit 5?

Problem 6. How many numbers between 1 and $30,000$ (inclusive) are there that are divisible by at least one of the numbers 6,10 and 30?

Problem 7. Show that the Fibbonaci's satisfy

$$F_1 + F_2 + \cdots + F_n = F_{n+2} - 1$$

for $n \geq 1$.