Problem 1. Find the GCD of the following pairs:
(a) 12135, 1213505
(b) $144^3$, $144^{35}$
(c) $144^3$, $144^4 + 6$

Problem 2. Find all integer solutions to
\[
\begin{align*}
x &\equiv 0 \pmod{13} \\
x &\equiv 4 \pmod{23} \\
x &\equiv 0 \pmod{103}
\end{align*}
\]

Problem 3. Find $x$ and $y$ between 0 and 1608 so that
\[
\begin{align*}
12x + y &\equiv 11 \pmod{1609} \\
x - y &\equiv 15 \pmod{1609}
\end{align*}
\]
(Note: 1909 is prime.)

Problem 4. Let $a_n$ and $b_n$ be sequences defined by
\[
\begin{align*}
a_1 &= 4, \quad a_2 = 1, \\
a_n &= a_{n-1} + 2a_{n-2} \\
b_1 &= 2, \quad b_2 = 3, \\
b_n &= b_{n-1} + 2b_{n-2}
\end{align*}
\]
For which $n$ is $a_n \geq b_n$?

Problem 5.
(a) How many integers between 10,000 and 99,999 (inclusive) are even and do not contain the digit 5?
(b) How many integers between 10,000 and 99,999 (inclusive) are odd and do not contain the digit 5?

Problem 6. How many numbers between 1 and 30,000 (inclusive) are there that are divisible by at least one of the numbers 6, 10 and 30?

Problem 7. Show that the Fibonacci's satisfy
\[
F_1 + F_2 + \cdots + F_n = F_{n+2} - 1
\]
for $n \geq 1$. 