HOMEWORK #6

Each problem requires a proof. I want these written carefully. You can try reading your solutions out loud. If you are saying lots of words not on the page, add some of those in.

Problem 1. Section 5.1, Number 6(e).

Problem 2. Show by induction that, for \( n \geq 1 \),

\[
\sum_{k=1}^{n} (3k-1)^2 = \frac{1}{2} n (6n^2 + 3n - 1).
\]

That is, show

\[
2^2 + 5^2 + 8^3 + \cdots + (3n-1)^2 = \frac{1}{2} n (6n^2 + 3n - 1)
\]

Problem 3. Suppose we define two sequences \( G_n \) and \( H_n \) by

\[ G_1 = 1, \ G_2 = 3, \]

and

\[ G_n = G_{n-1} + G_{n-2} \quad (n \geq 3) \]

and

\[ H_1 = 6, \ H_2 = 8, \]

and

\[ H_n = H_{n-1} + H_{n-2} \quad (n \geq 3). \]

Show, by induction, that \( H_n = 2G_{n+1} \) for \( n = 1, 2, 3, \ldots \)

Problem 4. Suppose we define two sequences \( G_n \) and \( H_n \) by

\[ G_1 = 1, \ G_2 = 3, \]

and

\[ G_n = G_{n-1} + G_{n-2} \quad (n \geq 3) \]

and

\[ H_1 = 2, \ H_2 = 2, \]

and

\[ H_n = H_{n-1} + H_{n-2} \quad (n \geq 3). \]

For which \( n \) is \( G_n \) less than or equal to \( H_n \)? (Prove your answer is correct. I suggest induction.)