HOMEWORK #6

Each problem requires a proof. I want these written carefully. You can try reading your solutions out loud. If you are saying lots of words not on the page, add some of those in.

Problem 1. Section 5.1, Number 6(e).

Problem 2. Show by induction that, for $n \geq 1$,

$$\sum_{k=1}^{n}(3k-1)^2 = \frac{1}{2}n\left(6n^2+3n-1\right).$$

That is, show

$$2^2+5^2+8^3+\cdots+(3n-1)^2 = \frac{1}{2}n\left(6n^2+3n-1\right)$$

Problem 3. Suppose we define two sequences $G_n$and H_n by

$$G_1 = 1,\ G_2 = 3,$$

and

$$G_n = G_{n-1} + G_{n-2} \quad (n \geq 3)$$

and

$$H_1 = 6,\ H_2 = 8,$$

and

$$H_n = H_{n-1} + H_{n-2} \quad (n \geq 3).$$

Show, by induction, that $H_n = 2G_{n+1}$ for $n = 1, 2, 3, \ldots$

Problem 4. Suppose we define two sequences G_n and H_n by

$$G_1 = 1,\ G_2 = 3,$$

and

$$G_n = G_{n-1} + G_{n-2} \quad (n \geq 3)$$

and

$$H_1 = 2,\ H_2 = 2,$$

and

$$H_n = H_{n-1} + H_{n-2} \quad (n \geq 3).$$

For which n is G_n less than or equal to H_n? (Prove your answer is correct. I suggest induction.)