HOMEWORK #2

Problem 1. Consider the partition
\[ P = \{\{1, 3, 6, 7, 8, 9\}, \{2, 5\}, \{4\}, \{10\}\}. \]
Let \( \sim \) be the associated equivalence relation.
(a) What is 2?
(b) Is it true that 7 \( \sim \) 4?
(c) List 4 elements of \( A \) so that \( a \not\sim b \) for any distinct \( a \) and \( b \) in your list.

Problem 2. The following table describes an equivalence relation. Find the associated set of ordered pairs that is this relation.

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<thead>
<tr>
<th></th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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</table>

Problem 3. There are 5 equivalence relations on the set \( \{0, 1, 2\} \). Find them all.

Problem 4. Here is a partition of \( \mathbb{N} \) into a bunch of finite sets:
\[ \{\{1\}, \{2, 3\}, \{4, 5, 6, 7\}, \{8, 9, 10, 11, 12, 13, 14, 15\}, \ldots\} \]
(Each subset is twice as big as the one before, and contains consecutive integers.)
Let \( \sim \) be associated equivalence relation.
(a) What is \( \mathbb{T} \)?
(b) What is \( \mathbb{T'} \)?
(c) Is 4 \( \sim \) 4?
(d) Is 9 \( \sim \) 100?
(e) What is \( 5 \cap \mathbb{T} \)?
(f) What is \( 5 \cap \mathbb{T'} \)?
(g) What is \( 5 \cap \mathbb{U} \)?

Problem 5. Give an example of a relation on three elements that is reflexive, symmetric and antisymmetric.