HOMEWORK #5

A list of primes can make life easier. I have found http://primes.utm.edu/lists/small/1000.txt to be accurate. Remind me to put a list of primes on the next midterm.

If the web is not available to you, recall you need to test \( n \) by seeing if it is divisible by the numbers up to \( \sqrt{n} \), but you skip any number you know is composite.

Memorizing a few primes helps:

\[ 2, 3, 5, 7, 11, 13, 17, 19, 23 \]

Knowing a few tests for divisibility makes life easier. For numbers given to you in base 10:

- \( n \) is divisible by 2 if the last digit is even.
- \( n \) is divisible by 3 if the sum of the digits is divisible by 3.
- \( n \) is divisible by 5 if the last digit is 0 or 5.
- \( n \) is divisible by 9 if the sum of digits is divisible by 9.
- \( n \) is divisible by 11 if the alternating sum of the digits is divisible by 1:

\[
11 \text{ divides } n = d_k d_{k-1} \cdots d_2 d_1 \iff 11 \text{ divides } d_k - d_{k-1} + \cdots \pm d_2 \mp d_1.
\]

**Problem 1.** Find another base where the last rule also works (where divisibility by \( 11_b \) can be tested using the alternating sum of the digits. An informal explanation of why this works in base-\( b \) for your choice of \( b \) will be ok.

**Problem 2.** Solve for \( x \) and \( y \).

\[
x + y \equiv 5 \pmod{2801}
\]
\[
x - y \equiv 1 \pmod{2801}
\]

**Problem 3.** Find the smallest natural number \( x \) for which

\[
x \equiv 1 \pmod{417}
\]
\[
x \equiv 5 \pmod{10}
\]

**Problem 4.** Suppose we define two sequences \( G_n \) and \( H_n \) by

\[ G_1 = 1, \ G_2 = 3, \]

and

\[ G_n = G_{n-1} + G_{n-2} \quad (n \geq 3) \]

and

\[ H_1 = 2, \ H_2 = 2, \]
and

\[ H_n = H_{n-1} + H_{n-2} \quad (n \geq 3). \]

For which \( n \) is \( G_n \) less than or equal to \( H_n \)? (Prove your answer is correct. I suggest induction.)

**Problem 5.** Problem 6(c) of Section 5.1.

**Problem 6.** Problem 9(g) of Section 5.1.