HOMEWORK #5

A list of primes can make life easier. I have found http://primes.utm.edu/lists/small/1000.txt to be accurate. Remind me to put a list of primes on the next midterm.

If the web is not available to you, recall you need to test n by seeing if it is divisible by the numbers up to $\lfloor\sqrt{n}\rfloor$, but you skip any number you know is composite.

Memorizing a few primes helps:

2,3,5,7,11,13,17,19,23

Knowing a few test for divisibility makes life easier. For numbers given to you in base 10:

- n is divisible by 2 if the last digit is even.
- n is divisible by 3 if the sum of the digits is divisible by 3.
- n is divisible by 5 if the last digit is 0 or 5.
- n is divisible by 9 if the sum of digits is divisible by 9.
- n is divisible by 11 if the altenating sum of the digits is divisible by 1 :

$$11 \text{ divides } n = \mathrm{d}_k\mathrm{d}_{k-1}\cdots\mathrm{d}_2\mathrm{d}_1 \Leftrightarrow 11 \text{ divides } d_k - d_{k-1} + \cdots + \pm d_2 \mp d_1.$$

Problem 1. Find another base where the last rule also works (where divisibility by 11_b can be tested using the altenating sum of the digits. An informal explanation of why this works in base-b for your choice of b will be ok.

Problem 2. Solve for x and y.

$$\begin{aligned} x+y &\equiv 5 \quad (\bmod 2801) \\ x-y &\equiv 1 \quad (\bmod 2801) \end{aligned}$$

Problem 3. Find the smallest natural number x for which

$$\begin{aligned} x &\equiv 1 \quad (\text{mod } 417) \\ x &\equiv 5 \quad (\text{mod } 10) \end{aligned}$$

Problem 4. Suppose we define two sequences $G_n$and H_n by

$$G_1 = 1,\ G_2 = 3,$$

and

$$G_n = G_{n-1} + G_{n-2} \quad (n \geq 3)$$

and

$$H_1 = 2,\ H_2 = 2,$$

and

$$H_n = H_{n-1} + H_{n-2} \quad (n \geq 3).$$

For which n is G_n less than or equal to H_n? (Prove your answer is correct. I suggest induction.)

Problem 5. Problem 6(c) of Section 5.1.

Problem 6. Problem 9(g) of Section 5.1.