HOMEWORK #3

I guess I'll follow the book and declare that the natural numbers start at 1, so

$$\mathbb{N} = \{1, 2, 3, \ldots\}.$$

Problem 1. Consider the following partial order on $\mathbb{N} \times \mathbb{N}$:

$$(a_1, a_2) \leq (b_1, b_2) \quad \text{means} \quad a_1 < b_1 \text{ (or } a_1 = b_1 \text{ and } a_2 \leq b_2) .$$

(a) Does $\preceq$ have any maximal elements?
(b) Describe all the elements that are a common upper bound for $(2, 4)$ and $(1, 2)$. (That is, find all x such that $(2, 4) \preceq x$ and $(1, 2) \preceq x$.)
(c) Is there a least upper bound for $(3, 4)$ and $(1, 5)$? If so, what is it?

Problem 2. Consider the following partial order on $\mathbb{Z}$:

$$a \preceq b \quad \text{means} \quad a = b \text{ or } a^2 < b^2.$$

(a) Does $\preceq$ have a minimum element?
(b) Is there a least upper bound for 5 and -5. If so, what is it?

Problem 3. Can a relation be both an equivalence relation and a function?

Problem 4. Suppose

$$f : \{1, 2, 3, 4\} \to \{5, 6, 7\}.$$

If

$$f(1) = 5, \ f(2) = 6,$$

consider the possible solutions in x and y to

$$x = f(3) \text{ and } y = f(4)$$

that make f onto. How many different solutions are there? List them.

Problem 5. Suppose

$$f : \{1, 2, 3, 4\} \to \{5, 6, 7, 8, 9\}.$$

If

$$f(1) = 5, \ f(2) = 6,$$

consider the possible solutions in x and y to

$$x = f(3) \text{ and } y = f(4)$$

that make f one-to-one. How many different solutions are there? List them.

Problem 6. If $f : \mathbb{Z} \to \mathbb{Z}$ is defined by

$$f(x) = \left\lfloor \frac{3x}{2} \right\rfloor,$$

what is the range of f? Show that f is one-to-one.