I guess I’ll follow the book and declare that the natural numbers start at 1, so 
\[ \mathbb{N} = \{1, 2, 3, \ldots\} \, . \]

**Problem 1.** Consider the following partial order on \( \mathbb{N} \times \mathbb{N} \) :
\[ (a_1, a_2) \leq (b_1, b_2) \quad \text{means} \quad a_1 < b_1 \quad \text{(or} \quad a_1 = b_1 \text{ and} \quad a_2 \leq b_2) \, . \]

(a) Does \( \leq \) have any maximal elements?
(b) Describe all the elements that are a common upper bound for \((2, 4)\) and \((1, 2)\). (That is, find all \( x \) such that \((2, 4) \leq x \) and \((1, 2) \leq x\).)
(c) Is there a least upper bound for \((3, 4)\) and \((1, 5)\)? If so, what is it?

**Problem 2.** Consider the following partial order on \( \mathbb{Z} \) :
\[ a \preceq b \quad \text{means} \quad a = b \quad \text{or} \quad a^2 < b^2 \, . \]

(a) Does \( \preceq \) have a minimum element?
(b) Is there a least upper bound for 5 and \(-5\)? If so, what is it?

**Problem 3.** Can a relation be both an equivalence relation and a function?

**Problem 4.** Suppose 
\[ f : \{1, 2, 3, 4\} \rightarrow \{5, 6, 7\} \, . \]
If 
\[ f(1) = 5, \quad f(2) = 6, \]
consider the possible solutions in \( x \) and \( y \) to 
\[ x = f(3) \quad \text{and} \quad y = f(4) \]
that make \( f \) onto. How many different solutions are there? List them.

**Problem 5.** Suppose 
\[ f : \{1, 2, 3, 4\} \rightarrow \{5, 6, 7, 8, 9\} \, . \]
If 
\[ f(1) = 5, \quad f(2) = 6, \]
consider the possible solutions in \( x \) and \( y \) to 
\[ x = f(3) \quad \text{and} \quad y = f(4) \]
that make \( f \) one-to-one. How many different solutions are there? List them.
Problem 6. If $f : \mathbb{Z} \to \mathbb{Z}$ is defined by

$$f(x) = \left\lfloor \frac{3x}{2} \right\rfloor,$$

what is the range of $f$? Show that $f$ is one-to-one.