SECOND REVIEW, MATH 327

Problem 1. Find the general solution to the recurrence relation

$$a_n = 4a_{n-1} - 5a_{n-2} + 2a_{n-3}.$$

Problem 2. If, for all $n \geq 0$,

$$a_n = A2^n + B3^n + n$$

and

$$a_0 = 2$$

and

$$a_2 = 6,$$

what are A and B?

Problem 3. Suppose

$$a_1 = 1,$$
$$a_2 = 2$$

and, for $n \geq 2$,

$$a_n = 2a_{n-1} + a_{n-2}.$$

For which n is a_n even? Prove your answer.

Problem 4. Solve

$$\begin{aligned} 2x + y &\equiv 3 \pmod{4681} \\ 3x - y &\equiv 1 \pmod{4681} \end{aligned}$$

Problem 5. Show that for all $n \geq 4$,

$$F_n F_{n-3} - F_{n-1} F_{n-2} = (-1)^n.$$

Problem 6. How many ways can I distribute 20 identical marbles between four labeled boxes if I am obligated to put in an even number of marbles in each box?

Prove

$$\binom{n}{k} = \binom{n-2}{k} + 2\binom{n-2}{k-1} + \binom{n-2}{k-2}$$

Problem 7. Find integers u and v so that

$$u \cdot 399 + v \cdot 753 = \mathrm{GCD}(399, 753).$$