MATH 327, HOMEWORK #5

Problem 1. Show that for all integers $n \geq 1$,

$$\sum_{j=1}^{n} 3j^2 - 3j = n^3 - n.$$

Problem 2. Number 6(e) on page 156.

Problem 3. Show that for the Fibonacci numbers F_n, we have for all $n \geq 1$, that

$$F_1 + F_2 + \cdots + F_n = F_{n+2} - 1$$

Problem 4. Find all integer solutions to

$$\begin{aligned} x &\equiv 2 \pmod{2711} \\ x &\equiv 3 \pmod{27} \end{aligned}$$

Problem 5. Find all integer solutions to

$$\begin{aligned} 2x + 3y &\equiv 2 \pmod{631} \\ 3x + 2y &\equiv 3 \pmod{631} \end{aligned}$$

You can use the fact that 631 is a prime.

Problem 6. Find a closed form solution to the recurrence relation

$$\begin{aligned} a_0 &= 2 \\ a_1 &= 2 \\ a_n &= 5a_{n-1} - 6a_{n-2} \end{aligned}$$

Problem 7. Find a closed form solution to the recurrence relation

$$\begin{aligned} a_1 &= 8 \\ a_2 &= 20 \\ a_n &= 4a_{n-1} - 4a_{n-2} \quad (n \geq 3) \end{aligned}$$

Problem 8. For which n does 3 divide the Fibonnaci F_n? (You need not give a formal proof to validate your answer, but do need to supply some reasoning as to why your answer is correct.)