# MATH 327 HOMEWORK #4

**Problem 0.1.** Do §4.4 #4(b)(d)(e)

**Problem 0.2.** Solve the following congruences:

(1) $6x \equiv 6 \pmod{18}$
(2) $6x \equiv 7 \pmod{18}$
(3) $6x \equiv 6 \pmod{5^{100}}$

**Problem 0.3.** Solve the following simultaneous congruences:

$$\begin{aligned} x + y &\equiv 1 \pmod{1001} \\ x - y &\equiv 1 \pmod{1001} \end{aligned}$$

**Problem 0.4.** Solve the following simultaneous congruences for $x$ and $y$ in terms of the constant integers $a$ and $b$:

$$\begin{aligned} x + y &\equiv a \pmod{1001} \\ 2x - y &\equiv b \pmod{1001} \end{aligned}$$

**Problem 0.5.**

(1) Using the modified Euclidean algorithm (c.f. page 108 or lecture notes) to find an integral linear combination of 17 and 41 that equals 1

$$u17 + v41 = 1 \quad (m, n \in \mathbb{Z})$$

(2) Find all solutions to

$$17x \equiv 1 \pmod{41}$$

(3) Find all solutions to

$$41x \equiv 1 \pmod{17}$$

**Problem 0.6.**

(1) Using the equation

$$13 \times 55 = 21 \times 34 + 1$$

(or $13 \times 55 + (-21) \times 34 = 1$) find one solution to the following simultaneous congruences:

$$\begin{aligned} x &\equiv 0 \pmod{55} \\ x &\equiv 1 \pmod{34} \end{aligned}$$

(2) We can rewrite $13 \times 55 = 21 \times 34 + 1$ as

$$(-21) \times 34 = (-13) \times 55 + 1.$$

Use this to find one solution to

$$\begin{aligned} x &\equiv 1 \pmod{55} \\ x &\equiv 0 \pmod{34} \end{aligned}$$

(3) Now add appropriate multiples of your two solutions to find one solution to

$$\begin{aligned} x &\equiv 2 \pmod{55} \\ x &\equiv 3 \pmod{34} \end{aligned}$$

**Problem 0.7.** Do §5.1 #6(e).

**Problem 0.8.** Do §5.1 #10(c).

**Problem 0.9.** Do §5.2 #1(b)(d).