Here is code we claim will return, for each nonnegative integer n, the equivalent binary representation.

We use `&` to indicate concatenation, so for example

```
"Strong"&"Induction" = "StrongInduction"
```

We use $\lfloor x \rfloor$ to indicate rounding down to an integer, so that

$$\lfloor \pi \rfloor = 3 \quad \lfloor -\pi \rfloor = -4 \quad \lfloor 12 \rfloor = 12$$

```
1)      function X(integer n) returns String
2)      {
3)          if n = 0 return "0";
4)          else if n = 1 return "1";
5)          else{
6)              q = ⌊n/2⌋;
7)              r = n − 2q;
8)              if r = 0 return X(q)&"0";
9)              else if r = 1 return X(q)&"1";
10)          }
11)     }
```

We can prove this code is correct using strong induction.

We need to have two base cases, since $X()$ handles both $n = 0$ and $n = 1$ as special cases. For $n = 0$, the condition on line (3) holds true, so the function returns the string `"0"`, which is correct. For $n = 1$ the condition on line (3) is false, so line (4) executes. Since the condition there is true, the function returns `"1"`, which is correct.

Now assume the function returns the correct value if it has any input in the range $0, 1, \ldots, k$, where $k \geq 1$. Now suppose the program is called on $n = k + 1$. Notice that $n \geq 2$ which means nothing will happen on lines (3) and (4) and execution will continue on line (6). After it assigns a value to q, either

$$2q = n \text{ or } 2q = n - 1$$

and in either case,

$$0 \leq q \leq \frac{n}{2} \leq n - 1 = k.$$

After line (7) assigns a value to r, we will have

$$n = 2q + r$$

and

$$r = 0 \text{ or } r = 1$$

If $r = 0$ the condition on line (8) is satisfied, and the function X will be called on q. We know $0 \leq q \leq k$ and so we also know that the function returns the correct value here, i.e., $\mathrm{X}(q)$ will be the binary representation of q. What is returned is that followed by a zero, which is the binary representation for $2q$, but $2q = 2q + r = n$ so this is correct.

If $r = 1$ nothing happens on line (8), but the condition on line (9) will be true. We know that $\mathrm{X}(q)$ will return the binary representation of q. What is returned is that followed by a one, which is the binary representation of $2q + 1$, and $2q + 1 = 2q + r = n$, so this is correct.