1) Let
\[ f(z) = \int_0^z (1 + \zeta)^{-1/2}(1 - \zeta)^{-1/2} d\zeta . \]
Determine the image of the real line under the map
\[ z \rightarrow f(z) . \]

Note: One has to be careful with the definition of the roots. Choose the root
\[ g_1(\zeta) = (1 - \zeta)^{-1/2} \]
so that \( g_1(\zeta) > 0 \) for \( \zeta < 1 \) and so that the function \( g_1 \) is holomorphic in \( \mathbb{H} \) and continuous in \( \mathbb{H} \setminus \{1\} \). Similarly for
\[ g_2(\zeta) = (1 + \zeta)^{-1/2} . \]

2) Let
\[ f(z) = \int_1^z (\zeta + 1)^{-2/3}(\zeta - 1)^{-2/3} d\zeta . \]
Show that \( f \) maps the extended real line \( (\mathbb{R} \cup \{\infty\}) \) onto a triangle. What kind of triangle?

3) Fix an angle \( \alpha \) with \( 0 < \alpha < \pi \) and consider the function
\[ f(z) = \int_0^z \zeta^{-(\pi - \alpha)/\pi} d\zeta . \]
a) Describe the image of the real line under the map \( z \rightarrow f(z) \).
   b) In this case it is not too difficult to describe the image of the open upper half-plane \( \mathbb{H} \) under \( f \). You could parametrize the straight line from 0 to \( z \) by
   \[ \zeta(t) = zt, \quad 0 \leq t \leq 1 , \]
   for example. Describe the image of \( \mathbb{H} \) under the map \( f \).
   What do you obtain?

4) Let \( z_j = 2^j \) for \( j = 0, 1, 2, \ldots \) and let
\[ U = \mathbb{C} \setminus \{z_0, z_1, z_2, \ldots\} . \]
Prove that the series
\[ f(z) = \sum_{j=0}^{\infty} \left( \frac{2^j}{z - 2^j} + 1 \right) \]

converges normally in \( U \).

If \( f(z) \) a meromorphic function?