1) Let $S$ denote the open half-strip
\[ S = \{ z = x + iy : -\frac{\pi}{2} < x < \frac{\pi}{2}, y > 0 \} . \]

a) Determine the image of $S$ under the map $z \rightarrow \sin z$.

b) Determine the image of the boundary $\partial S$ of $S$ under the map $z \rightarrow \sin z$.

c) Is the function $f(z) = \sin z$ one-to-one on $\overline{S} = S \cup \partial S$?

2) Let
\[ w(q) = \int_0^q (1 + \zeta)^{-1/2}(1 - \zeta)^{-1/2} d\zeta \]
for $q \in \mathbb{R}$. How do you have to define the roots so that the function $q \rightarrow w(q)$ for $q \in \mathbb{R}$ inverts the function $z \rightarrow \sin z$ on $\partial S$?

3) Let
\[ f(z) = \int_1^z (\zeta + 1)^{-2/3}(\zeta - 1)^{-2/3} d\zeta . \]
Choose the roots as in class. Determine the image of the extended real line $\mathbb{R} \cup \{ \infty \}$ under the map $z \rightarrow f(z)$.

4) Fix an angle $\alpha$ with $0 < \alpha < \pi$ and consider the function
\[ f(z) = \int_0^z \zeta^{-(\pi - \alpha)/\pi} d\zeta . \]
Choose the root as in class.

a) Describe the image of the real line under the map $z \rightarrow f(z)$.

b) In this case it is not too difficult to describe the image of the open upper half-plane $\mathbb{H}$ under $f$. You can parametrize the straight line from 0 to $z$ by
\[ \zeta(t) = zt, \quad 0 \leq t \leq 1 , \]
for example. Describe the image of $\mathbb{H}$ under the map $f$.

5) Let $z_j = 2^j$ for $j = 0, 1, 2, \ldots$ and let
\[ U = \mathbb{C} \setminus \{ z_0, z_1, z_2, \ldots \} . \]

Prove that the series
\[ f(z) = \sum_{j=0}^{\infty} \left( \frac{2^j}{z - 2^j} + 1 \right) \]
converges normally in $U$.

If $f(z)$ a meromorphic function?