MATH 464/514, Applied Matrix Theory, Fall 2023

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Homework 6, assigned Nov. 17, due Dec. 1

1. Use hand computations to transform the following two matrices to Jordan normal form

$$A = \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 & 1 \\ -4 & 0 & -2 \\ -4 & 0 & -2 \end{pmatrix} .$$

2. a) Let

$$A = \begin{pmatrix} 7/5 & 1/5 \\ -1 & 1/2 \end{pmatrix} .$$

Determine $\lim_{n\to\infty} A^n$ by hand. Check your result using Matlab and compute A^n for large n.

b) Let

$$A = \frac{1}{2} \begin{pmatrix} 1+\alpha & -1+\alpha \\ -1+\alpha & 1+\alpha \end{pmatrix}$$

where α is a complex number. For which values of α does the limit $\lim_{n\to\infty} A^n$ exist? Determine the limit for the cases where it exists.

3. Let

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1.98 & .99 \\ -.99 & 0 \end{pmatrix} .$$

Transform these three matrices to Jordan normal form.

4. Consider the powers

$$A^j, \quad B^j, \quad C^j, \quad j = 1, 2, \ldots$$

for the three matrices of Problem 3. For which of the three matrices will the powers remain bounded? For which of the three matrices will the powers converge to the zero matrix as $j \to \infty$?

5. Use a computer to plot the matrix norms of the matrices

$$A^j, \quad B^j, \quad C^j, \quad j = 1, 2, \ldots$$

for a reasonable range of j values.

You may use Matlab's command $norm(M)$ to obtain a norm of the matrix M. The matrix norm $norm(M)$ computed by Matlab is the largest singular value of M. This is the matrix norm corresponding to which vector norm? Give a reason.