Midterm 2: Math 321
April 6, 2023, 9:30-10:45am

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
Total	120	

Instructions

- Print your name below and sign the statement. Exams which lack these will not be graded.
- Show your work on each problem, presenting it in a neat and organized manner.
- Give the solution to each problem in the space provided. If you need more space to answer a problem, use the back of a page, making a note where the additional work can be found.
- You are allowed both sides of a single 8.5" x 11" sheet of paper for notes. Hand your note sheet in with your exam.

Please sign and date the following statement:
I have read the instructions and understand them. I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Printed Name Signature Date

1. Let A be and $n \times k$ matrix and B a $k \times m$ matrix so that AB is a well-defined $n \times m$ matrix. Prove that if the columns of B are linearly dependent, then so are the columns of AB.

2. Determine if the given matrix A in each part is invertible.

   - If it is invertible, use an augmented matrix and row operations to find A^{-1} explicitly.
   - If it is not invertible, state the rank of A.

   (a) $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

3. (a) Prove that if $\mathbf{b} \neq \mathbf{0}$, then the set of solutions to $A\mathbf{x} = \mathbf{b}$ is not a subspace.

(b) Prove that the subset of $\mathbb{R}^3$ consisting of vectors of the form

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

such that $v_1 - 2v_2 + v_3 = 0$ defines a subspace of $\mathbb{R}^3$.

4. Let A and B be the 3×4 matrices

$$A = \begin{bmatrix} 1 & -2 & 0 & -4 \\ 2 & -4 & 1 & -5 \\ -1 & 2 & 1 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 & 0 & -4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

It can be checked that A and B are row equivalent, though you do not need to do this.

(a) Find a basis for $\text{null}(A)$ and state the dimension of $\text{null}(A)$.

(b) Find a basis for $\text{col}(A)$ and state the dimension of $\text{col}(A)$.

(c) Do the columns of A span $\mathbb{R}^3$? Justify your answer.

5. Expand the given set to form a basis for $\mathbb{R}^3$

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \right\}$$

6. Suppose A is an $n \times m$ matrix whose column space is m-dimensional. Prove that the columns of A are linearly independent.